

PHYSICS 140A--SOLID STATE PHYSICS
WINTER QUARTER, 2015-16
SYLLABUS

Instructor: Professor Charles S. Fadley, Physics 241
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- Course website: <http://140a.physics.ucdavis.edu> . Regularly updated throughout the quarter.
- Class meetings: Tuesdays and Thursdays, 10:30 AM - 11:50 AM, 140 Physics

- Textbooks

-- M. Ali Omar, "Elementary Solid-State Physics", Revised Printing, Addison-Wesley-Longman, 1993. Primary text for lecture, reading, and homework problems. Clearly written, although older. Out of print, but can be purchased in international edition for \$18.50 from:

<http://www.valorebooks.com/textbooks/elementary-solid-state-physics-principles-and-applications/9780201607338>

--H. Ibach and H. Luth, "Solid-State Physics: An Introduction to Principles of Materials Science", 4th edition, Springer, 2009. Much more up-to-date and advanced. Will be used for some reading assignments as well.

Available as a free download from the Library or at the course website for UCD students, or in paperback for about \$50 or less.

- A couple good websites for viewing crystal structures, with others as we go along:

<http://www.dawgsk.org/crystal/>

<http://www.fhi-berlin.mpg.de/KHsoftware/Balsac/pictures.html> (A downloadable program for creating your own structures)

- Course grading: Based on the following breakdown--

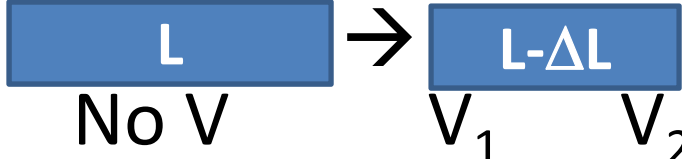
Graded problem sets:	25%
Midterm examination	25%
Final examination	50%
	100%

Exam crib sheet: You may bring in one 8.5" x 11" sheet with important equations and results written on it to both the midterm and the final.

- Tentative syllabus: Readings below are mostly from Omar, but complementary reading from Ibach and Luth will also be assigned throughout the quarter. Copies of viewgraphs and other supplementary material will be posted at the class website and will be an important component of study for the course and exams.

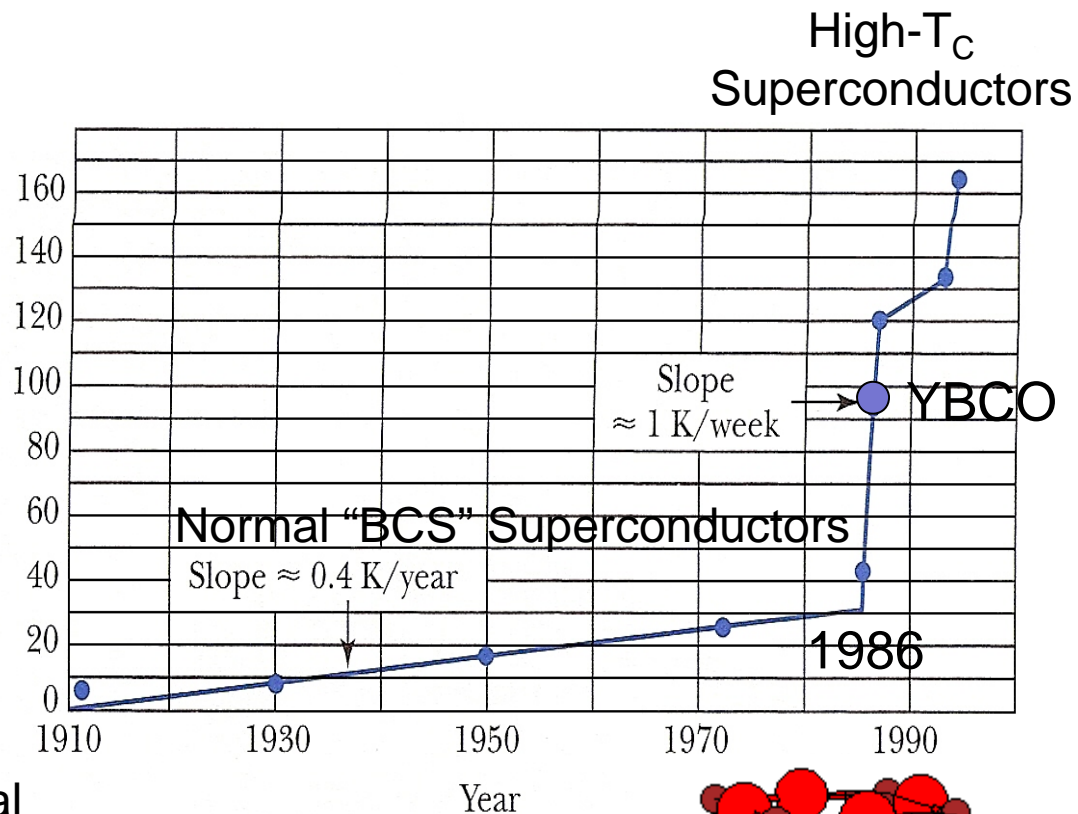
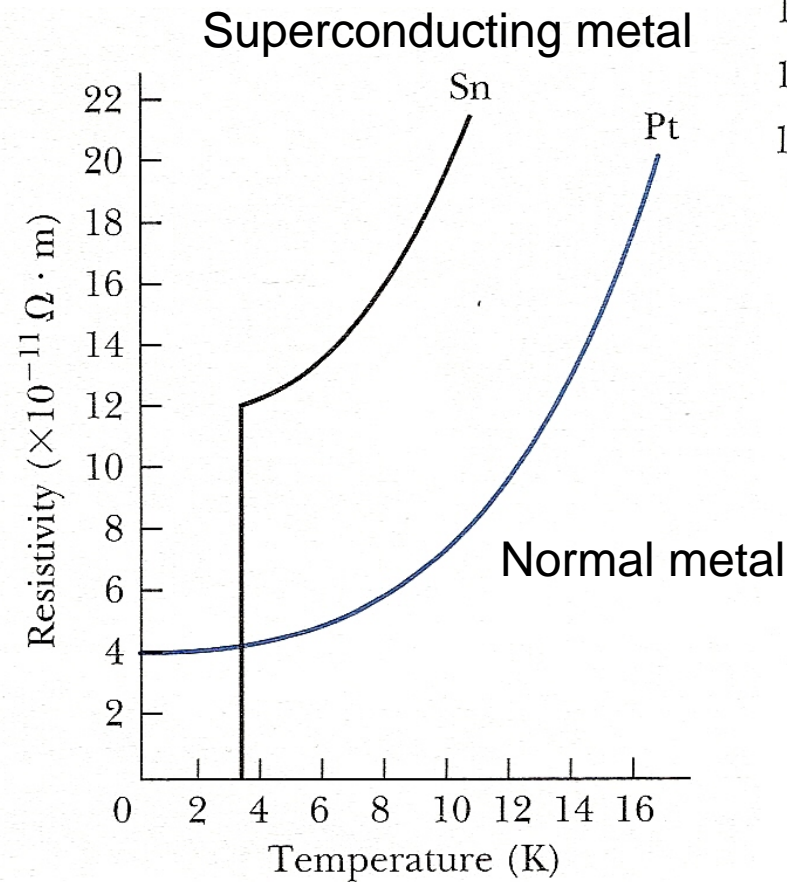
<u>Week</u>	<u>Dates</u>	<u>Topic [Reading: O = Omar, IL = Ibach and Luth]</u>
1	1/5-1/7	Course intro., Crystal bonding and structure [O-Ch.1, App A1-A8; IL-Ch. 1]
2	1/12-1/14	Crystal bonding and structure (continued)
3	1/19-1/21	Diffraction in Crystals [O-Ch. 2]
4	1/26-1/28	Diffraction (continued)
5	2/2-2/4	Lattice vibrations [O-Ch. 3]
6	2/9	Midterm Examination
	2/11	Lattice vibrations (continued)
7	2/16	Lattice vibrations (continued)
	2/18	The free-electron model [O-Ch. 4]
	--Approximately here, optional tour of Lawrence Berkeley National Laboratory solid state research facilities--	
8	2/23-2/25	The free-electron model (continued)
9	3/1-3/3	Energy bands in solids [O-Ch. 5]
10	3/8-3/10	Energy bands in solid (continued)
Final	3/19, Saturday	Final Examination: 10:30 AM-12:30 PM, Physics/Geology 140

Some properties of solids/condensed matter

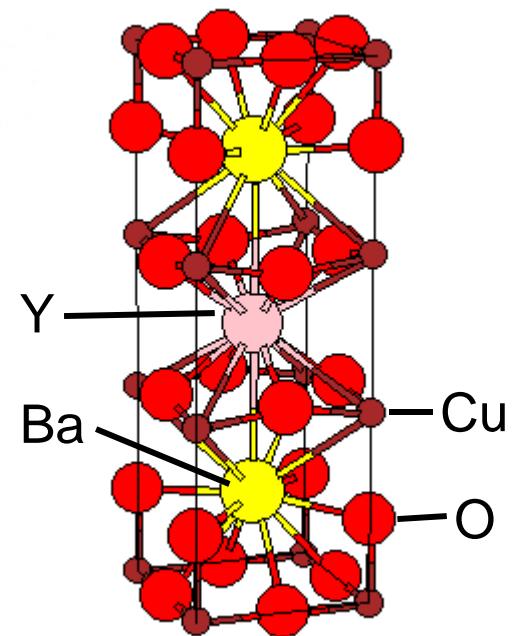
- Hard \rightarrow soft, ductile
- Conducting \rightarrow Semiconducting \rightarrow Insulating
- Superconducting: current with no voltage!
- Magnetic: Ferromagnetic (... $\uparrow\uparrow\uparrow\uparrow$...), Anti-ferromagnetic (... $\uparrow\downarrow\uparrow\downarrow$...), ...
- Piezoelectric: 

The diagram illustrates the piezoelectric effect. On the left, a blue rectangular block is labeled with the letter 'L' in white, representing its length. Below it, the text 'No V' indicates that no voltage is applied. An arrow points to the right, where the same blue rectangular block is shown, but its length is shorter, labeled 'L-ΔL' in white. Below this shorter block, two voltages are indicated: 'V₁' on the left and 'V₂' on the right, representing the applied electric field.
- Surprising/"Emergent" properties: Complex mixtures or multilayer nanostructures or nanoparticles
- And much more...

High temperature superconductivity

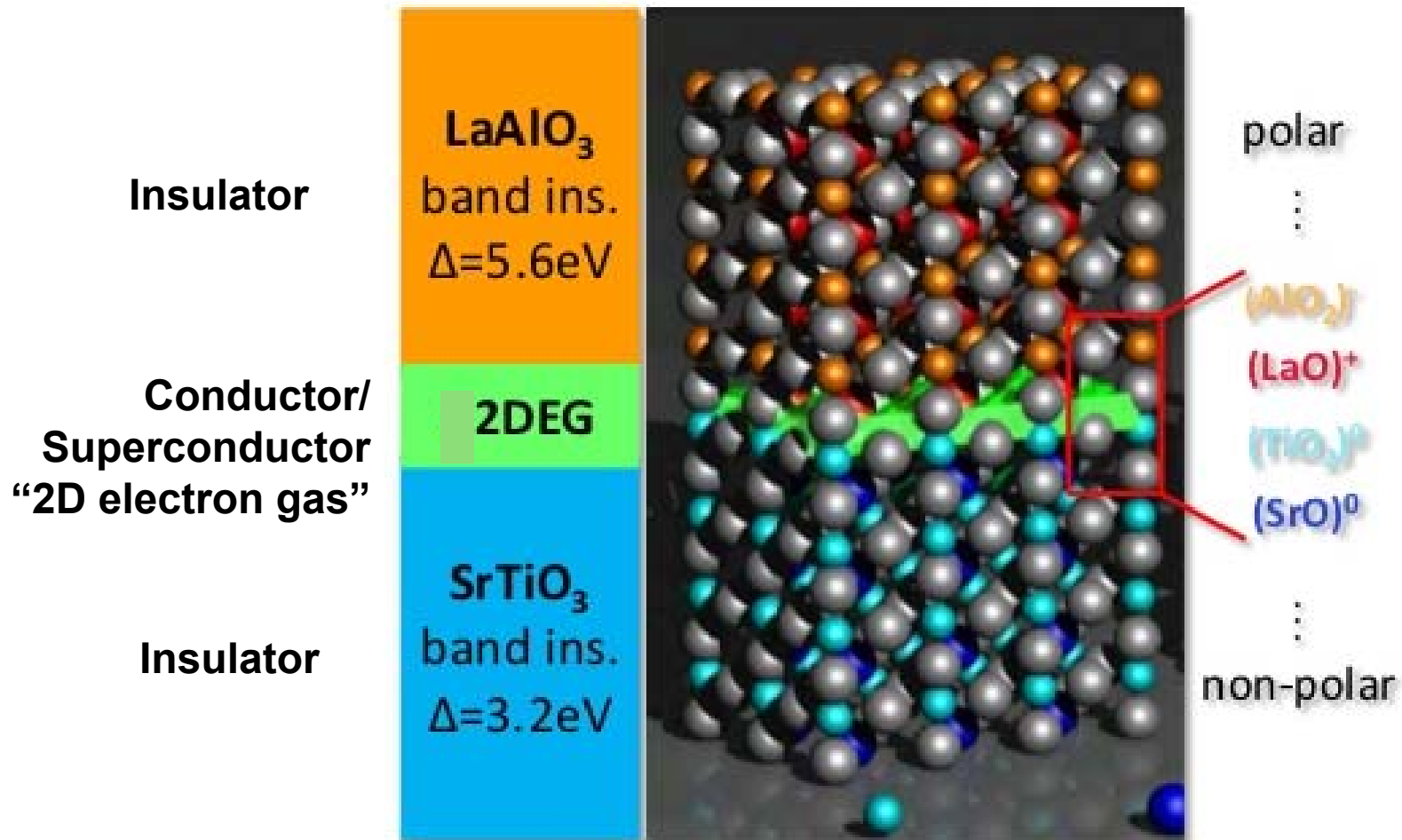


"High T_C ":
 $\text{YBa}_2\text{CuO}_{7-\delta}$
 = "YBCO"



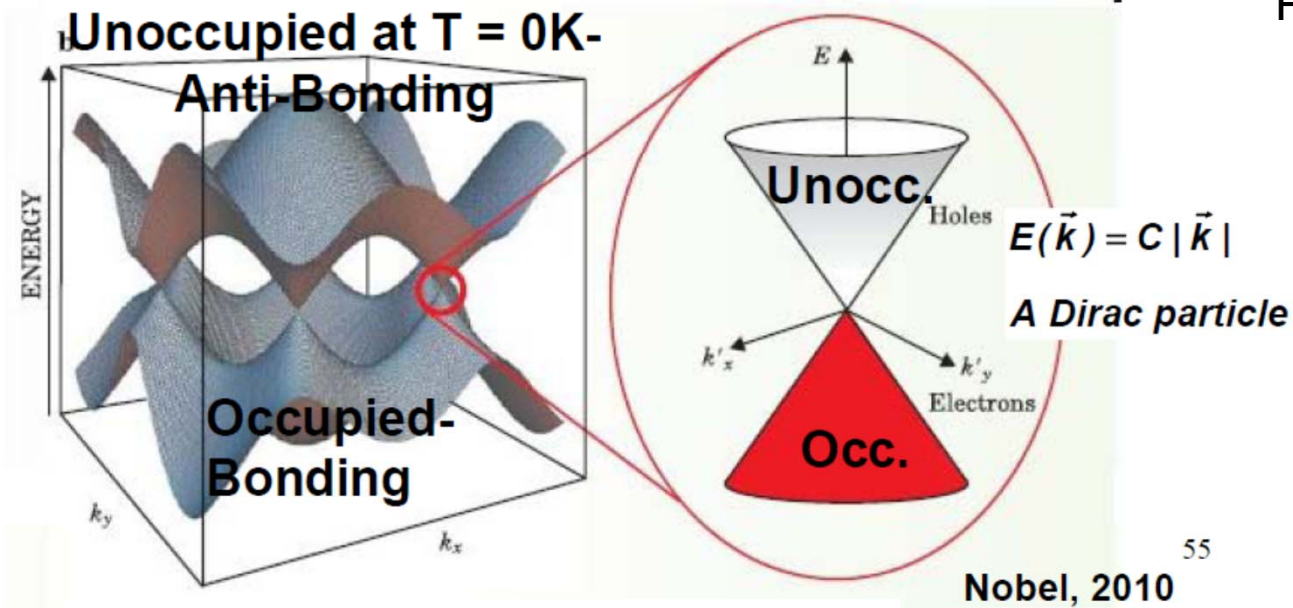
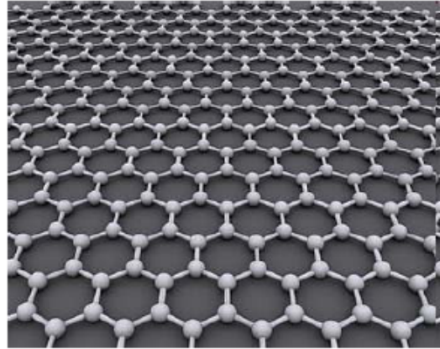
Nobel Prize in Physics 1987: Bednorz & Mueller: For their important breakthrough in the discovery of superconductivity in ceramic materials

Emergent properties at interfaces



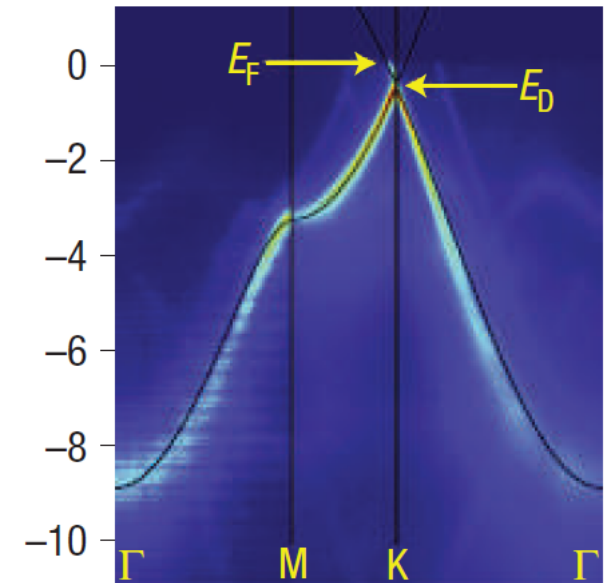
Ohtomo/Hwang, *Nature* **427**, 423 (2004)

Strange electrons in simple materials: graphene



The Nobel Prize in Physics 2010:
Geim, Novoselov, for groundbreaking
experiments regarding the two-
dimensional material graphene"

Photoelectron spectroscopy

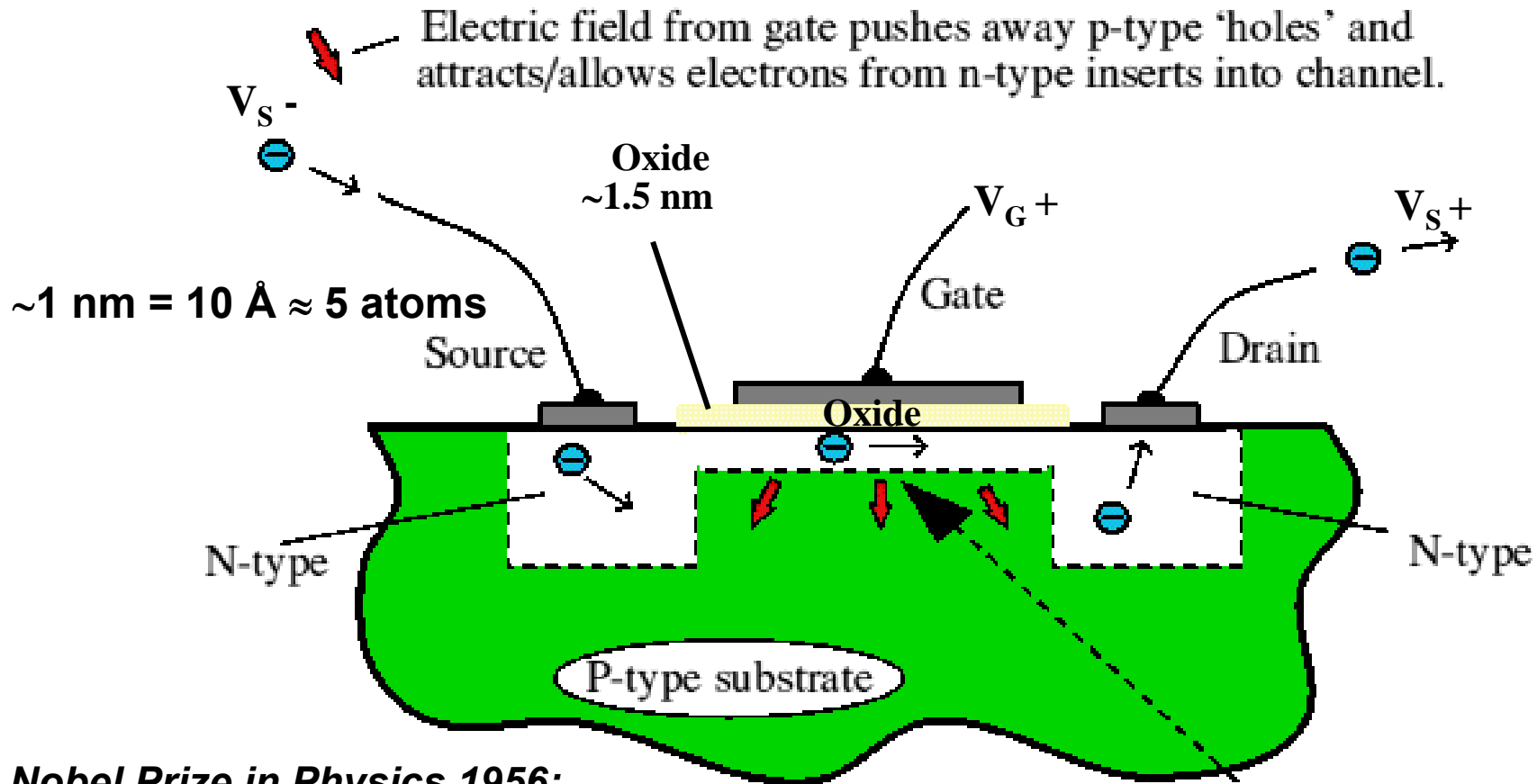


Slide Set 6 5

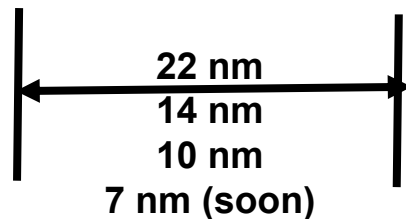
Bostwick et al., Nature Physics 3, 36 - 40 (2007)

**The IT workhorse:
the metal-oxide-semiconductor field-effect transistor (MOSFET)**

■ Metal contacts ■ Insulating layer



**Nobel Prize in Physics 1956:
Shockley, Bardeen, Bratrain, for
their researches on
semiconductors and their
discovery of the transistor effect**



Channel
~1.5 nm
n- or p-
type

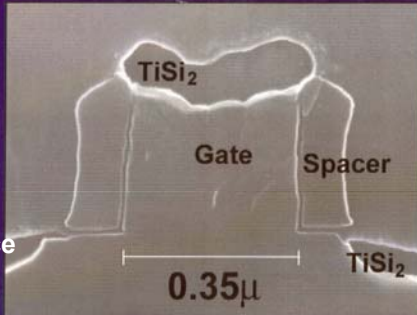
Transistors keep shrinking → Moore's Law



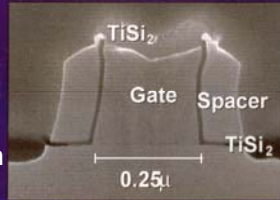
<http://www.intel.com/technology/mooreslaw/index.htm>

And the Shrink Goes On...

.35 μ Process Technology



.25 μ Process Technology



0.032 microns = 32 nm ('10) →
→ 22 nm (2012) → 14 nm (2014) →

High-k + Metal Gate Transistors

Metal Gate

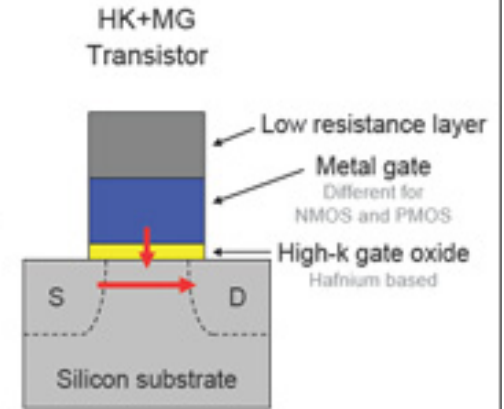
- Increases the gate field effect

High-k Dielectric

- Increases the gate field effect
- Allows use of thicker dielectric layer to reduce gate leakage

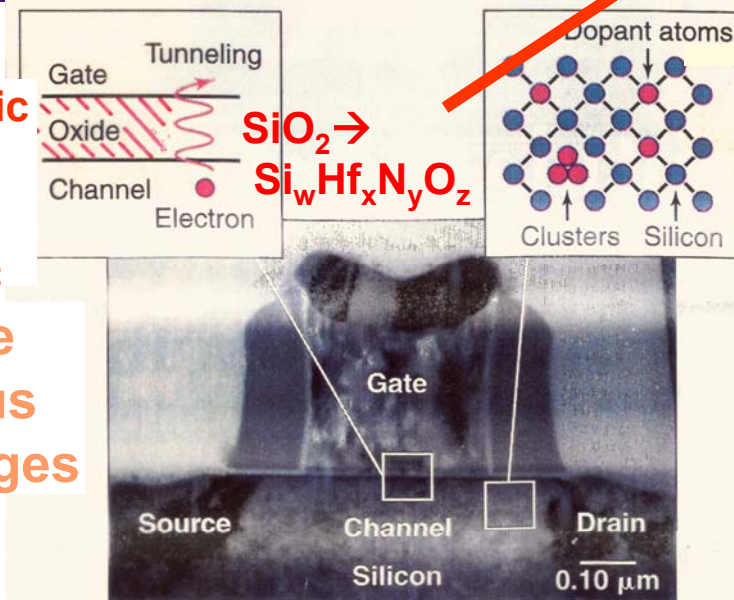
HK + MG Combined

- Drive current increased >20% (>20% higher performance)
- Or source-drain leakage reduced >5x
- Gate oxide leakage reduced >10x



~few atomic layers—
currently
15 Å SiO₂

Some serious challenges



~1 %

Cross section of a MOS transistor. Electron tunneling through the gate oxide (left inset) and high-concentration dopant interactions (right inset) are posing fundamental limitations to continuing historical transistor scaling trends.

- 10 μ m — 1971
- 3.0 μ m — 1975
- 1.5 μ m — 1982
- 1.0 μ m — 1985
- 800 nm — 1989
- 600 nm — 1994
- 350 nm — 1995
- 250 nm — 1997
- 180 nm — 1999
- 130 nm — 2002
- 90 nm — 2004
- 65 nm — 2006
- 45 nm — 2008
- 32 nm — 2010
- 22 nm — 2012
- 14 nm — 2014
- 10 nm — 2016
- 7 nm — est. 2018
- 5 nm — est. 2020

Current SiO_2 gate oxide thicknesses in the 1 nm range \rightarrow with high-k dielectrics, $\text{Si}_w\text{Hf}_x\text{N}_y\text{O}_z, \dots$ a few nm or more

Mixed oxides:
 Si^{+1} , Si^{+2} , Si^{+3} , and coord. sites

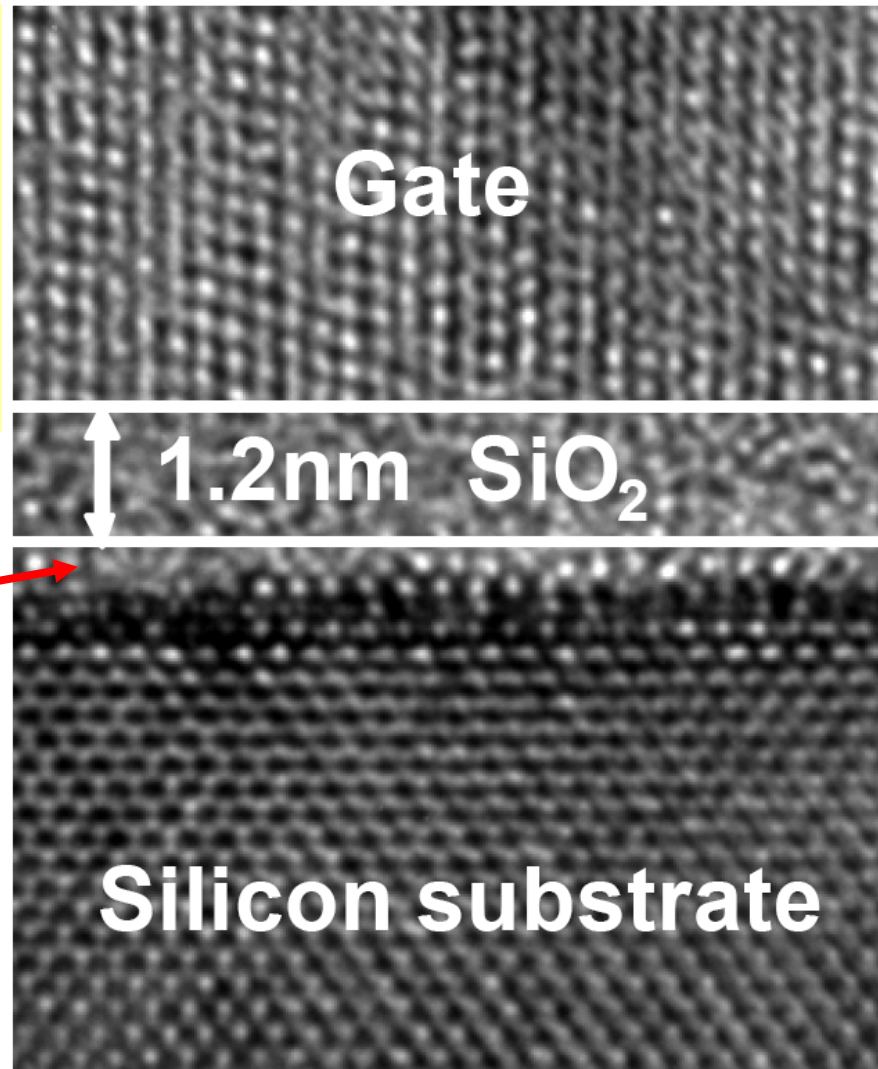
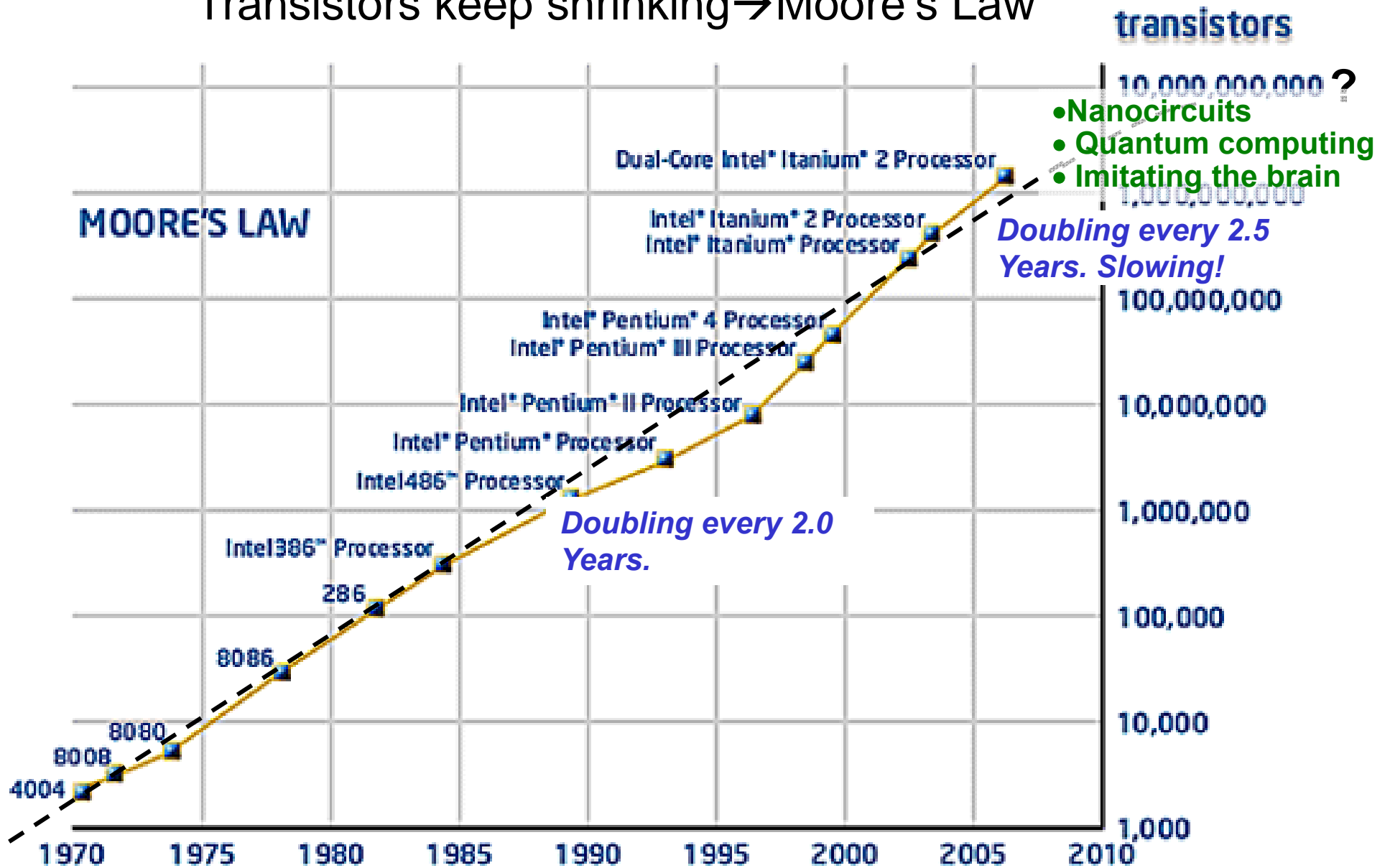


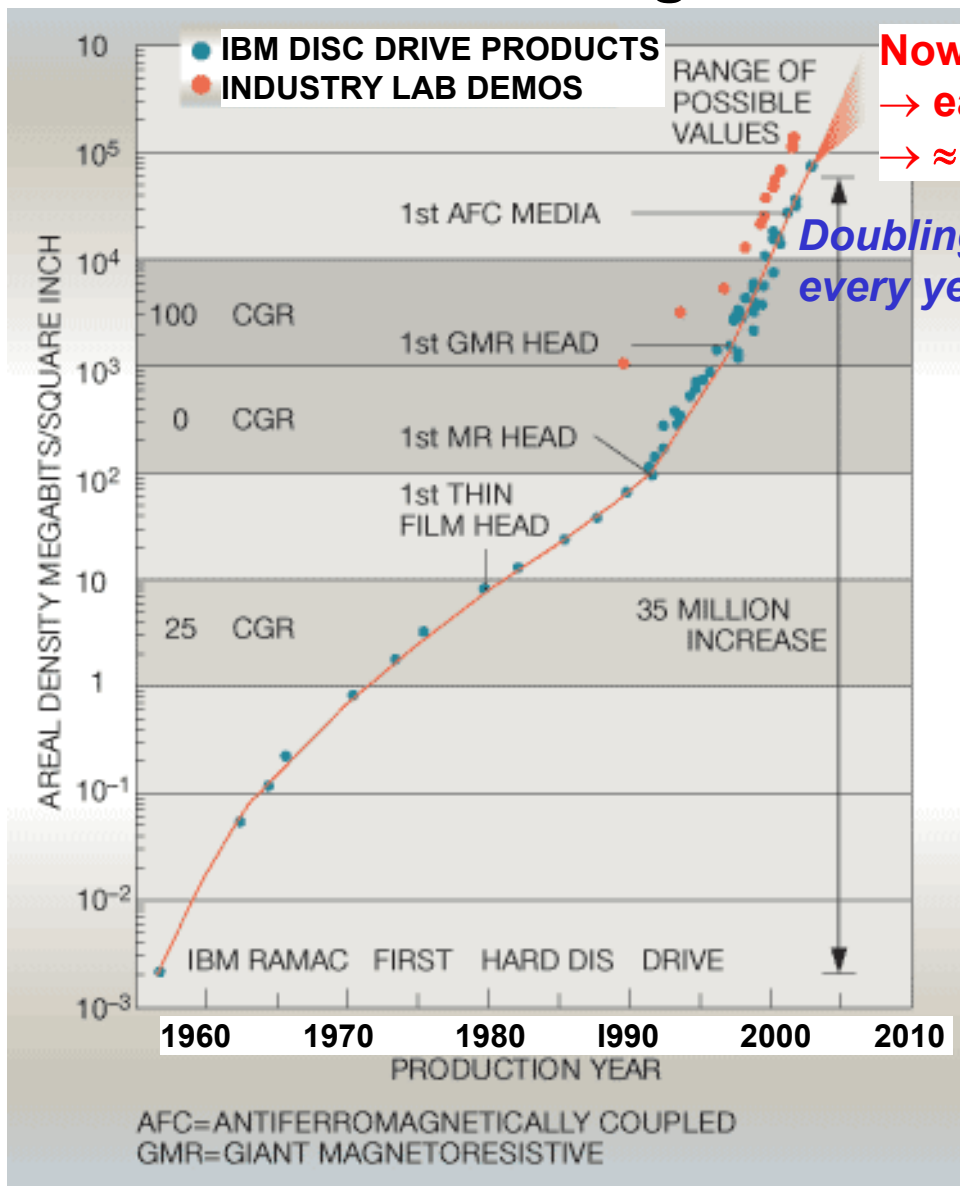
Fig. 2 High resolution TEM cross section of 1.2nm physical SiO_2 gate oxide at the 90nm logic technology node. \rightarrow 65 nm technology \rightarrow 45 nm \rightarrow 32 nm \rightarrow 22 nm \rightarrow 14 nm \rightarrow ...

Transistors keep shrinking → Moore's Law



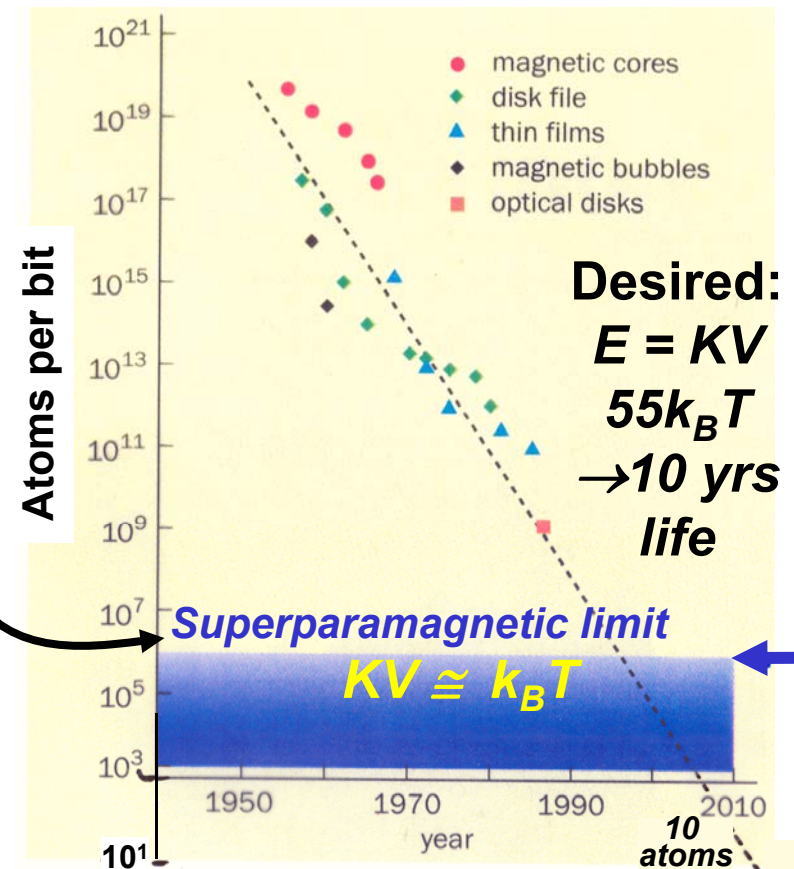
<http://www.intel.com/technology/mooreslaw/index.htm>

“Moore’s Law” for magnetic storage



Now 170 Gbits/in² = 20 Gbytes/in²
 → each bit ~14 nm x 30 nm x 210 nm
 → ≈ 4,000,000 atoms, read at GHz rates

How far can we go?

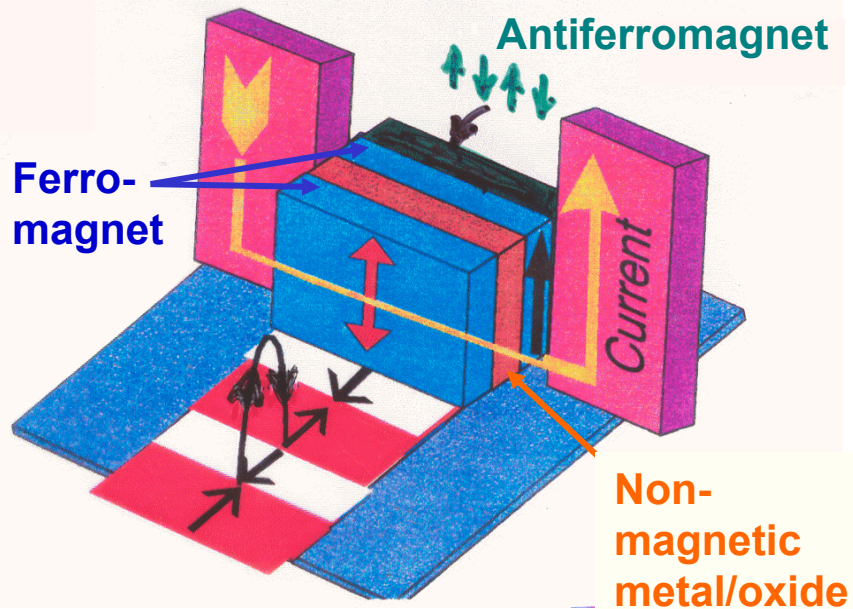


<http://www.research.ibm.com/journal/sj/422/grochowski.html>

Harris, Awschalom
 Physics World,
 Jan. '99

The number of atoms used to store one bit of information with different forms of magnetic or optical storage has reduced over the years. The blue region indicates the superparamagnetic regime, below which thermal fluctuations at room temperature could alter the orientation of magnetic bits.

Spin-Valve Read Head

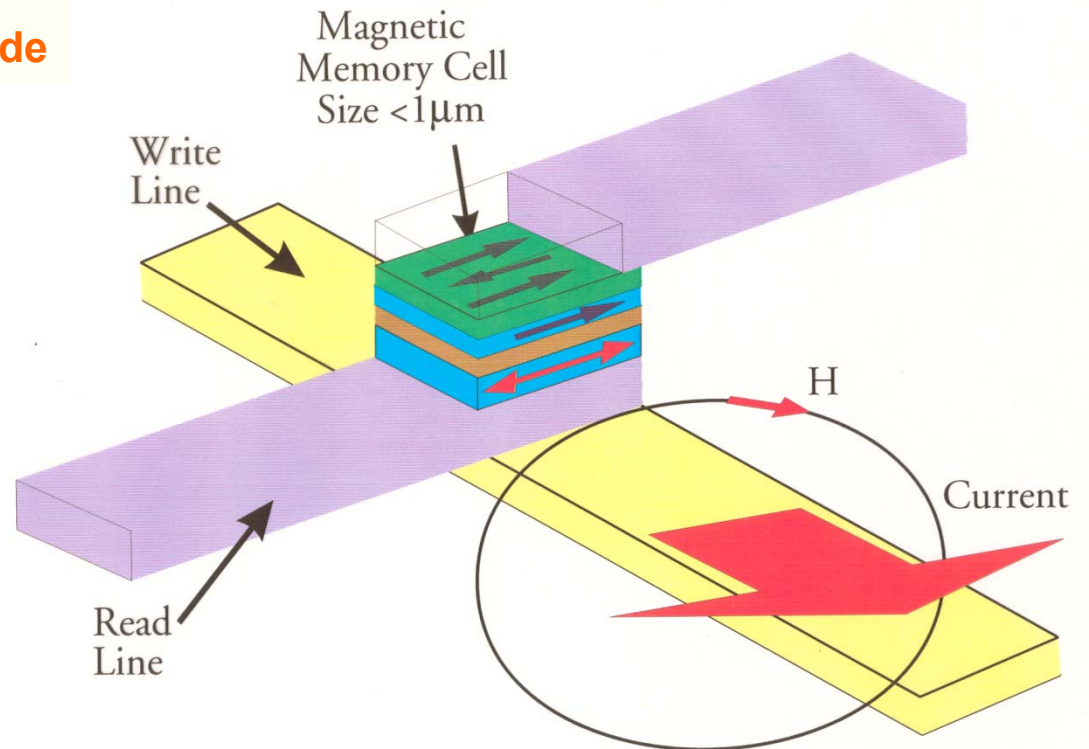


Giant change in resistance →
much smaller read heads

*Nobel Prize in Physics 2007: Fert
and Grünberg: for the discovery
of Giant Magnetoresistance*

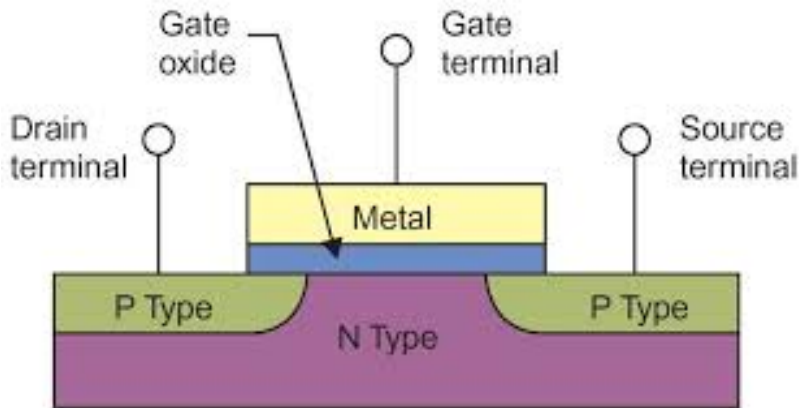
Some key elements in
Spintronics/Semiconductors/
Sensors—multilayer
nanostructures

Magnetic Random Access Memory (MRAM-Non Volatile)

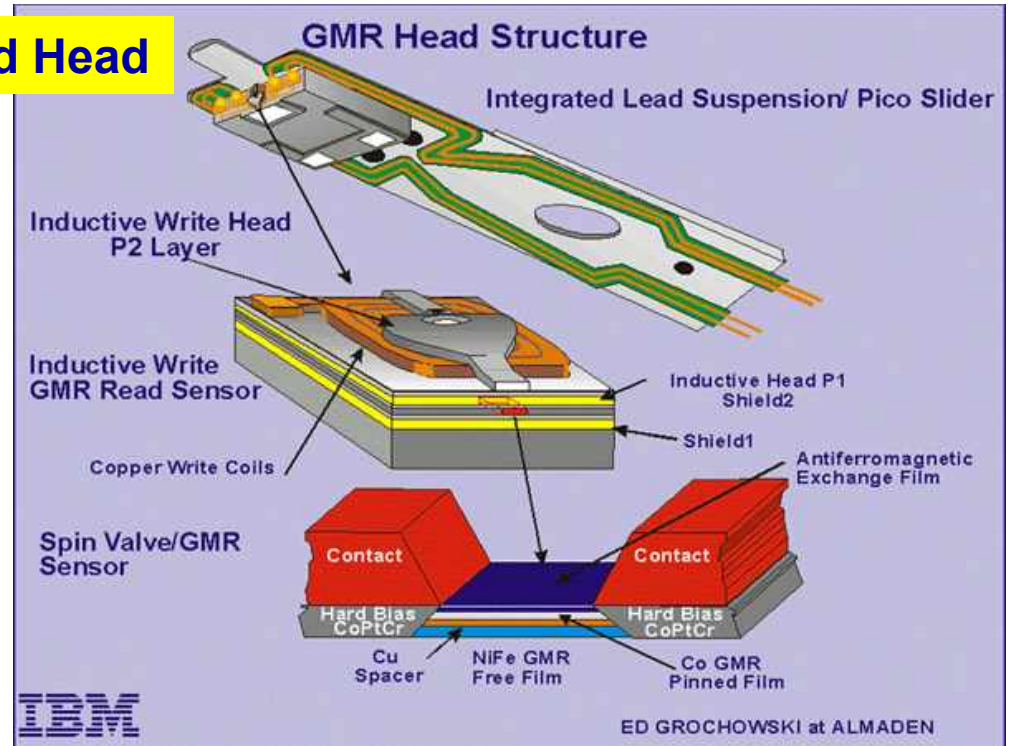


Multilayers and interfaces in information and energy technology

Transistor

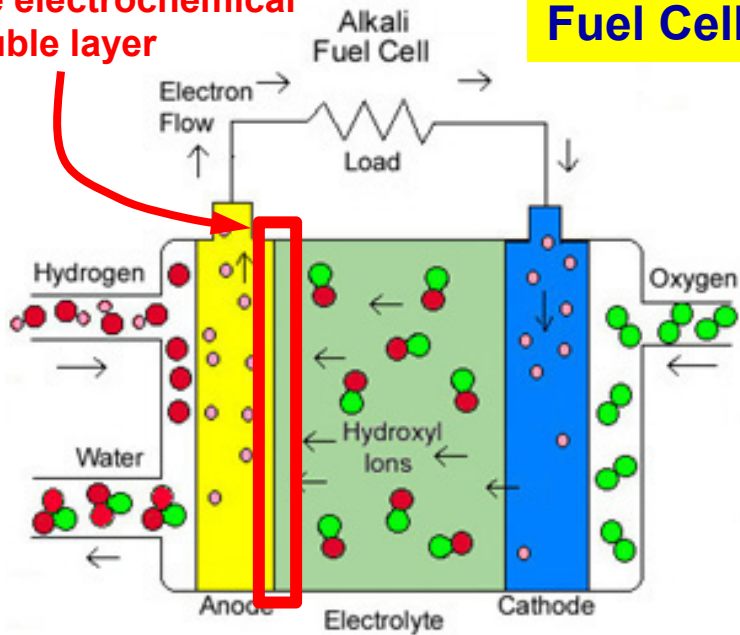


Hard Drive Read Head



The electrochemical double layer

Fuel Cell



Photovoltaic Cell

Electron and Current Flow in Solar Cells

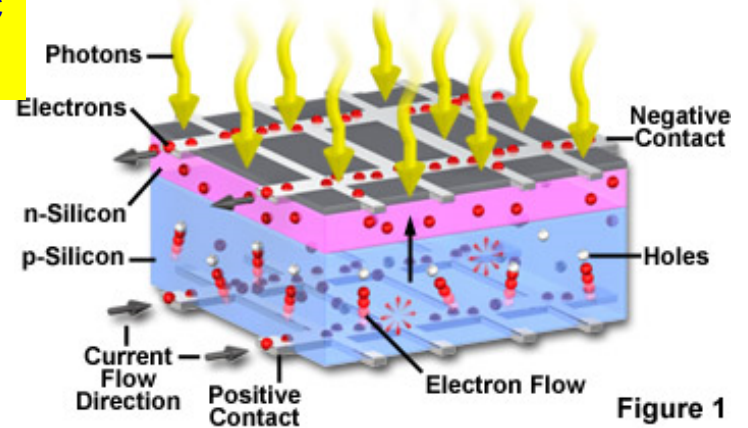
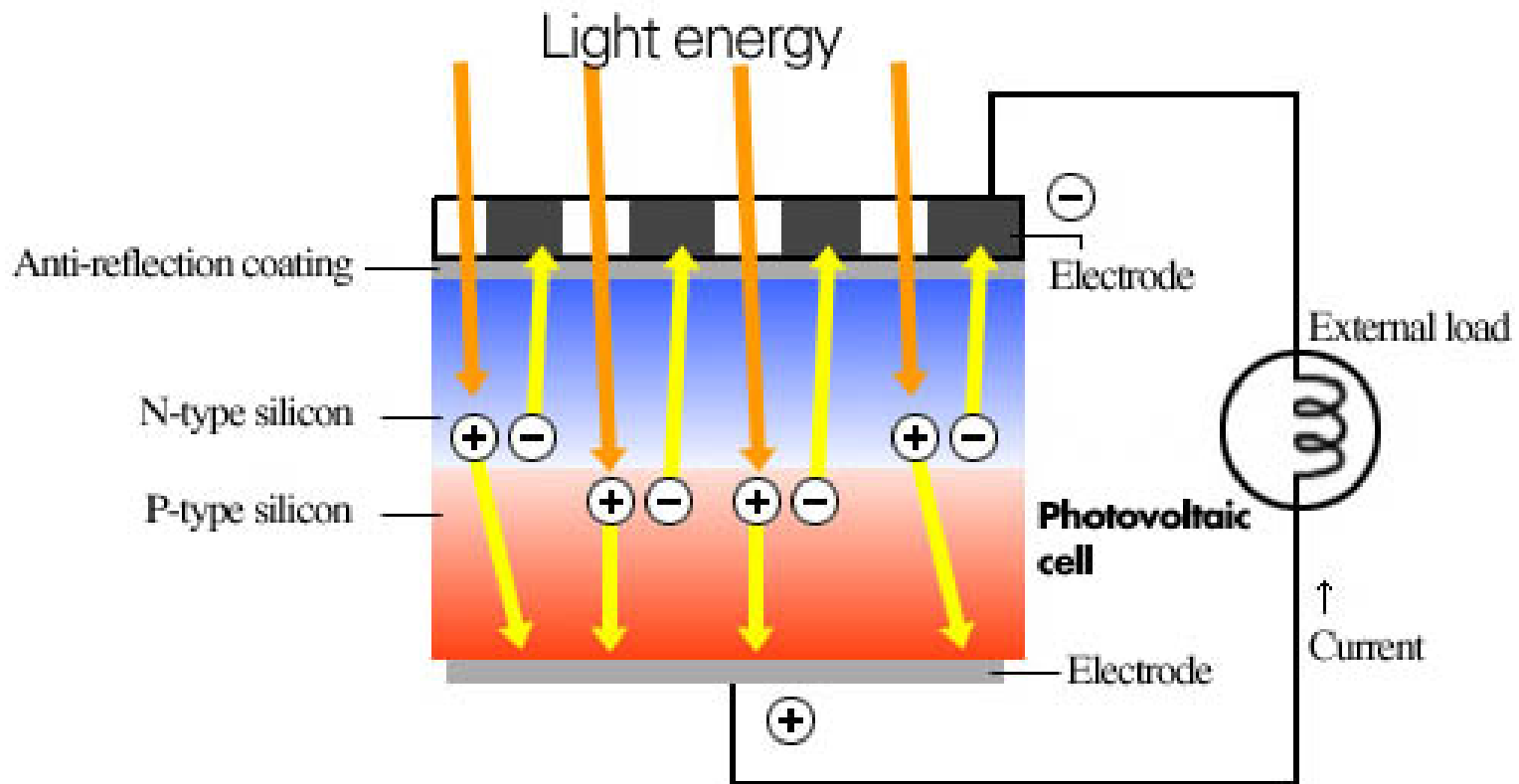
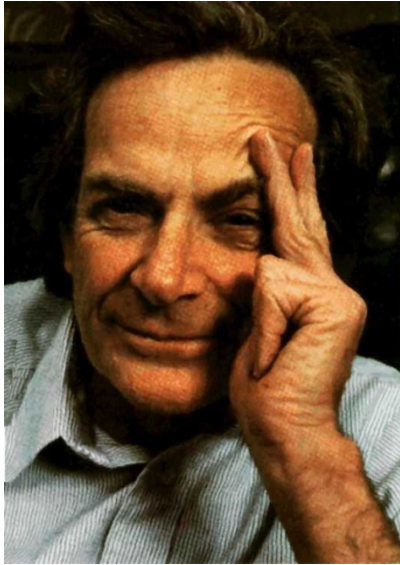


Figure 1

The photovoltaic cell

A photovoltaic cell generates electricity when irradiated by sunlight.





1959 — Richard P. Feynman
“There’s plenty of room
at the bottom”

“Why cannot we write the entire 24 volumes of the Encyclopedia Brittanica on the head of a pin?”

It would be possible if you could print it with dots (= bits) that are 8 nanometers or about 32 metal atoms across, containing about 1000 atoms, or a cube 10 x 10 x 10 atoms

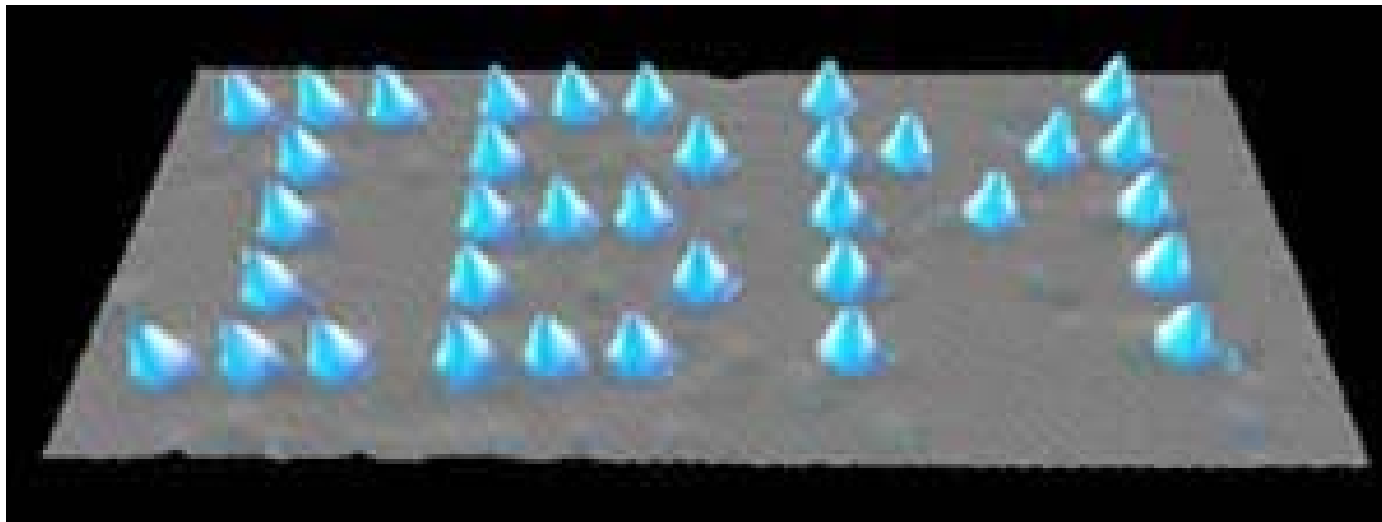
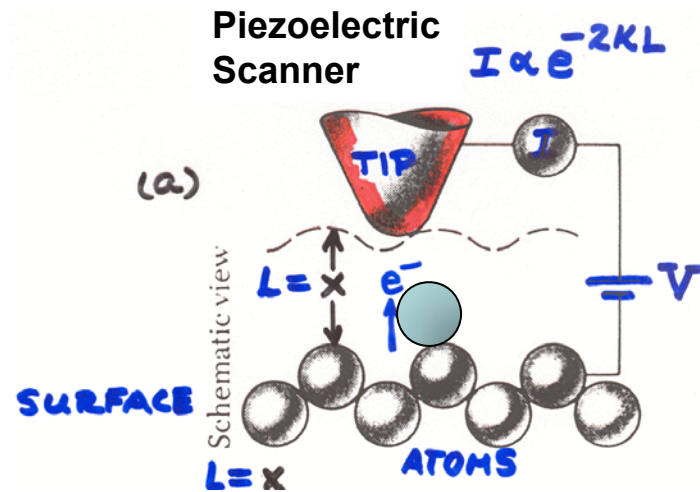
+ Many visionary ideas:

Miniaturization of computers

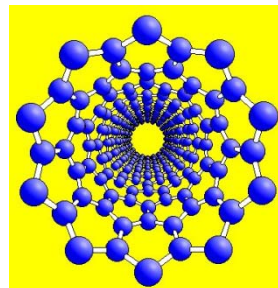
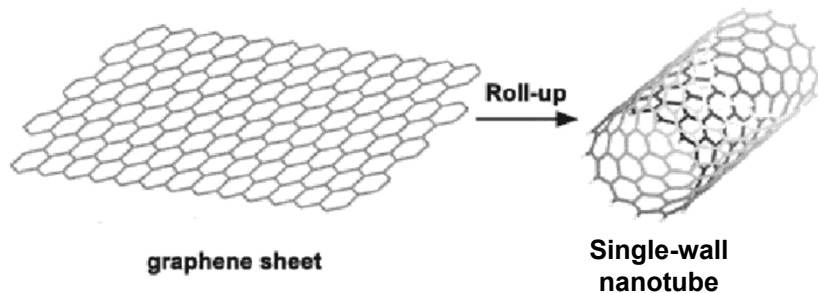
Imaging and manipulation of single atoms or molecules →→

<http://www.zyvex.com/nanotech/feynman.html>

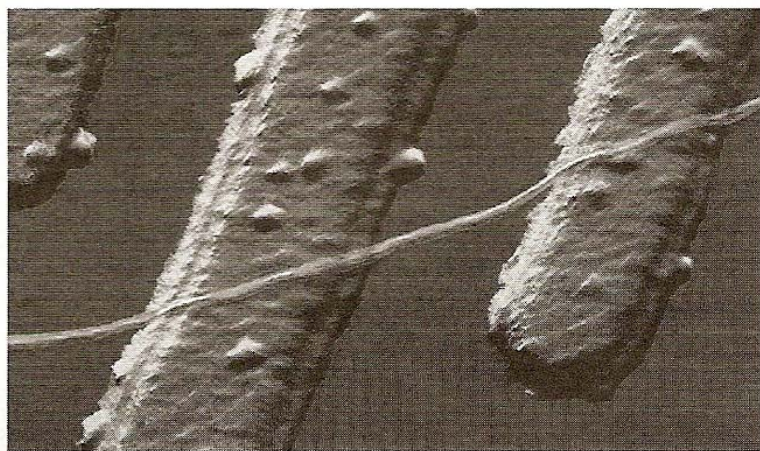
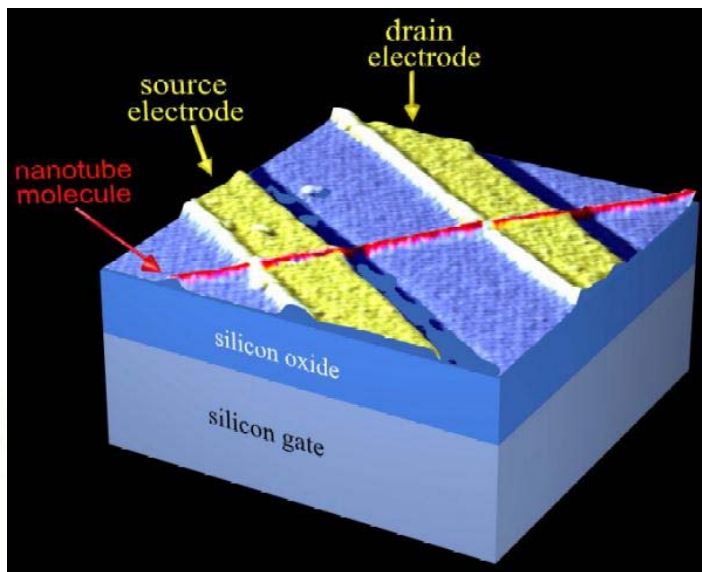
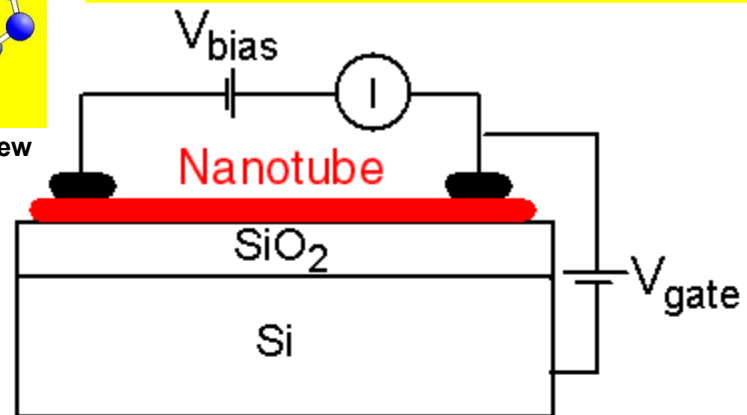
Writing with single atoms—30 years later



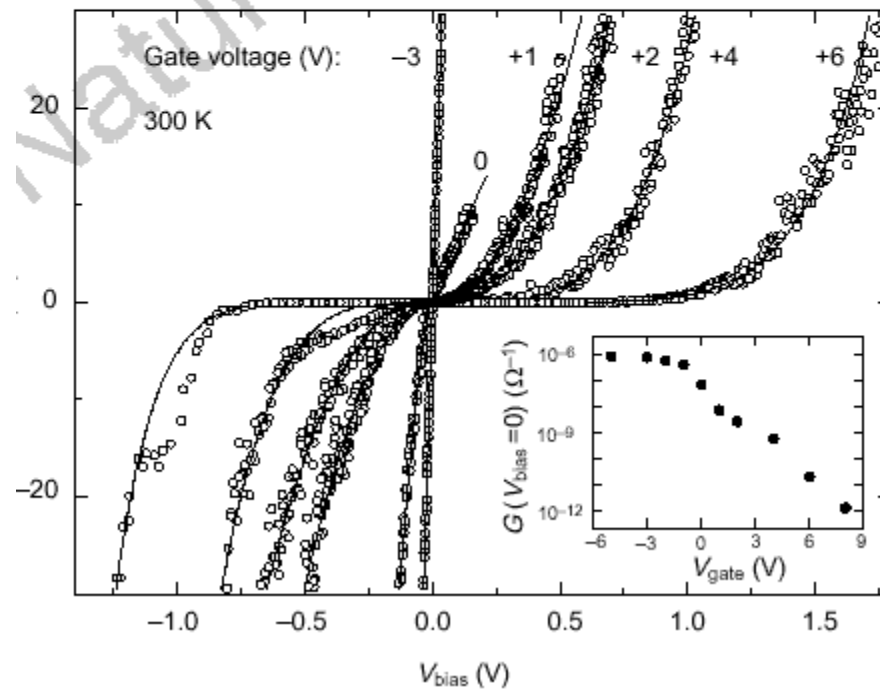
1989--IBM, written with single xenon atoms, using a scanning tunneling microscope, but too slow for real data storage—so far



A carbon nanotube transistor

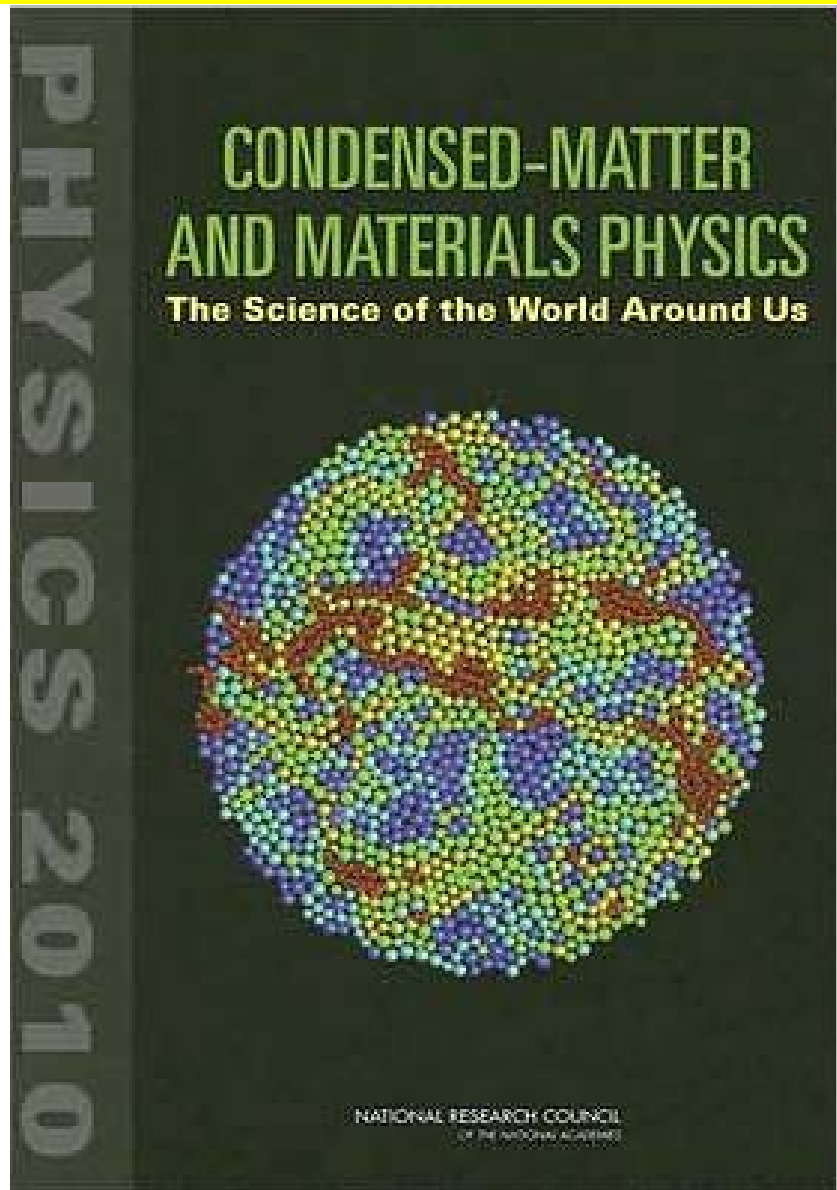


(b)



<http://ipn2.epfl.ch/CHBU/images/NTtransistir.gif>

A Look at the Future-- Condensed Matter Physics—The Science of the World Around Us



- *How Do Complex Phenomena Emerge from Simple Ingredients? →*
Strongly correlated materials
- *How Will the Energy Demands of Future Generations Be Met? →*
Solar cells, fuel cells,...
- *What New Discoveries Await Us in the Nanoworld? →*
Surfaces and interfaces, novel nanodevices
- *How Will the Information Technology Revolution Be Extended? →*
Nanoscale logic and memory, spin electronics = "spintronics"
- *What Happens Far from Equilibrium and Why? → Many nanoscale systems*
- *What Is the Physics of Life? →*
Biophysics

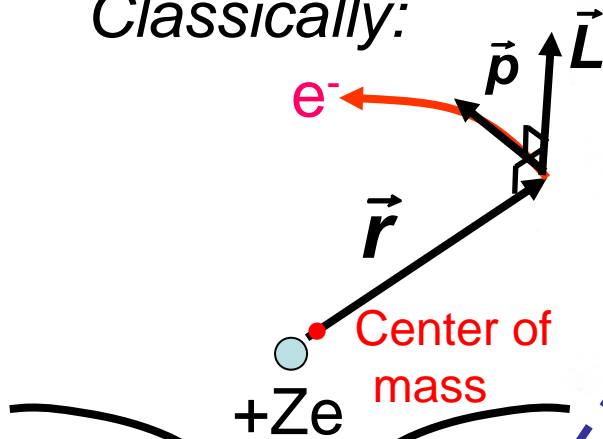
Publisher: National Academies Press

Pub. Date: December 2007

ISBN-13: 9780309109697, 286pp

The Hydrogenic Atom Schroedinger Equation: Spherical Polar Coordinates

Classically:



$V(r) = -Ze^2/4\pi\epsilon_0 r$

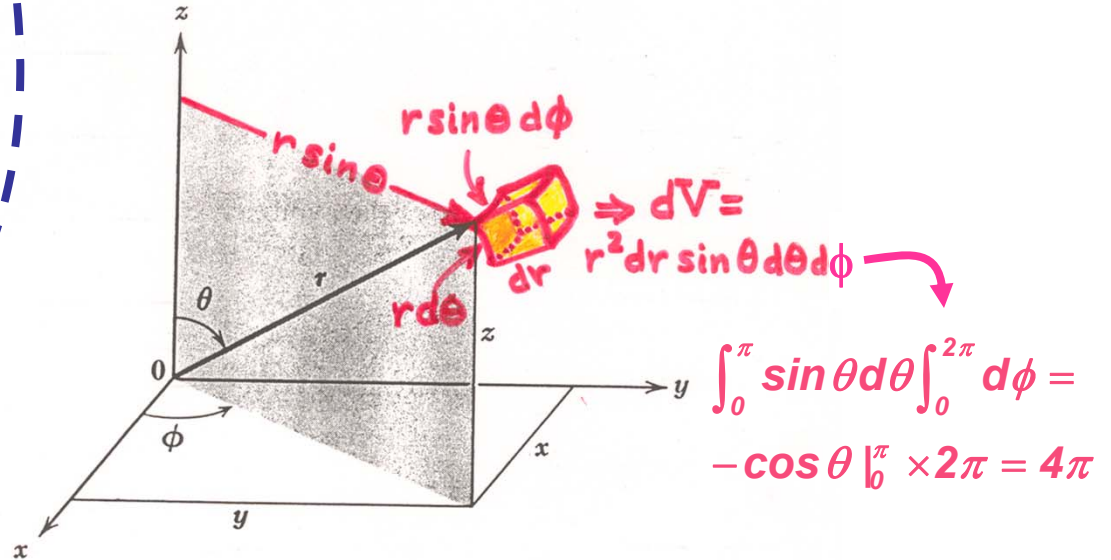
$\vec{L} = \vec{r} \times \vec{p}$

is conserved

$\mu = \text{reduced mass}$

$$= \frac{m_e}{1 + \frac{m_e}{M_n}}$$

Quantum mechanically:



$x = r \sin\theta \cos\phi$
 $y = r \sin\theta \sin\phi$
 $z = r \cos\theta$

Polar angle = $\theta = \arccos(z/r)$
 Azimuthal angle = $\phi = \arctan(y/x)$

$\hat{K} = -\frac{\hbar^2}{2\mu} \nabla^2 = -\frac{\hbar^2}{2\mu} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right]$ Converting to new coordinates

$$= -\frac{\hbar^2}{2\mu} \cdot \frac{1}{r^2 \sin\theta} \left[\sin\theta \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin\theta} \frac{\partial^2}{\partial \phi^2} \right]$$

$$= \frac{\hat{p}_r^2}{2\mu} + \frac{\hat{L}^2}{2\mu r^2}$$

$$= -\frac{\hbar^2}{2\mu} \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2} \cot\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \phi^2}$$

ψ H-ATOM SCH. EQN. IS:

$\hat{H}\psi(r, \theta, \phi) = \hat{K}\psi(r, \theta, \phi) - \frac{Ze^2}{4\pi\epsilon_0 r} \psi = E\psi$

$\rightarrow Z_{\text{eff}}(r)$ IN MANY-e ATOM

• USE SEPARATION OF VARIABLES :

$$\Psi(r, \theta, \phi) = R(r) \Theta(\theta) \Phi(\phi) \quad [\Theta(\theta) \equiv f(\theta) \text{ \& } \Phi(\phi) \equiv g(\phi) \text{ in text }]$$

- ASSUMED FORM -

• SUBSTITUTE, REARRANGE →

$$\phi: \frac{d^2 \Phi}{d\phi^2} + C_\phi^2 \Phi = 0 \Rightarrow \Phi(\phi) = A e^{\pm i C_\phi \phi} \quad \textcircled{1}$$

$$C_\phi = m_\ell = 0, \pm 1, \dots \quad \Phi_{m_\ell}(\phi) = \frac{1}{\sqrt{2\pi}} e^{\pm i m_\ell \phi} \quad \text{COMPLEX}$$

$$\theta: \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d\Theta}{d\theta} \right) + \left[C_\theta - \frac{m_\ell^2}{\sin^2 \theta} \right] \Theta = 0 \quad \textcircled{2}$$

$$\downarrow \quad \uparrow$$

$$l(l+1) \quad Z_{\text{eff}}(r) \text{ IN MANY } e^- \text{ ATOM}$$

$$R: \left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right) + \frac{l(l+1)}{2\mu r^2} - \frac{Z e^2}{4\pi \epsilon_0 r} \right] R = E R \quad \textcircled{3}$$

A "RADIAL SCHRÖDINGER EQN."

$$\hat{H}_{\text{radial}} R = E R \Rightarrow \begin{cases} E = E_n \\ = -\frac{Z^2 e^2}{8\pi \epsilon_0 a_0} \cdot \frac{1}{n^2} \\ n = 1, 2, 3, \dots \end{cases}$$

The Bohr Formula!

• SOLVING FOR Θ WITH $\textcircled{2} \Rightarrow$

$$\Theta_{l m_\ell}(\theta) = \text{ASSOC. LEGENDRE POLYNOMIALS IN } (\cos \theta) - \text{REAL}$$

$$l = 0, 1, 2, 3, \dots (n-1)$$

$$m_\ell = \underbrace{-l, -l+1, \dots, 0, \dots, +l-1, +l}_{2l+1}$$

The same for many- e^- atoms

SOLVING FOR R WITH $\textcircled{3} \Rightarrow$

$$R_{nl}(r) = \text{ASSOC. LAGUERRE FUNCTIONS ALSO } \equiv (\text{POLYNOMIAL IN } r) \cdot e^{-Zr/a_0} - \text{REAL}$$

Changes for many- e^- atoms

The atomic orbitals:

$$Y_{\ell m_\ell}(\theta, \phi) = \text{"spherical harmonics"}$$

COMPLEX, IF $m \neq 0$ But we can make them real for convenience

TABLE 6.1

NORMALIZED WAVE FUNCTIONS OF THE HYDROGEN ATOM FOR $n = 1, 2,$ AND 3^* ($Z=1 = \text{HYDROGEN}$)

n	ℓ	m_ℓ	$\Phi_{m_\ell}(\phi)$	$\Theta_{\ell m_\ell}(\theta)$	$R_{n\ell}(r)$	$\Psi_{n\ell m_\ell}(r, \theta, \phi) = \Phi_{m_\ell} \Theta_{\ell m_\ell} R_{n\ell}$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2} a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos \theta$	$\frac{1}{2\sqrt{6} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos \theta$
2	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin \theta$	$\frac{1}{2\sqrt{6} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin \theta e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3} a_0^{3/2}} \left(27 - 18 \frac{r}{a_0} + 2 \frac{r^2}{a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi} a_0^{3/2}} \left(27 - 18 \frac{r}{a_0} + 2 \frac{r^2}{a_0^2}\right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos \theta$	$\frac{4}{81\sqrt{6} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \cos \theta$
3	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin \theta$	$\frac{4}{81\sqrt{6} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \sin \theta e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4} (3 \cos^2 \theta - 1)$	$\frac{4}{81\sqrt{30} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} (3 \cos^2 \theta - 1)$
3	2	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{15}}{2} \sin \theta \cos \theta$	$\frac{4}{81\sqrt{30} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin \theta \cos \theta e^{\pm i\phi}$
3	2	± 2	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4} \sin^2 \theta$	$\frac{4}{81\sqrt{30} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin^2 \theta e^{\pm 2i\phi}$

*The quantity $a_0 = 4\pi\epsilon_0\hbar^2/me^2 = 5.3 \times 10^{-11} \text{ m}$ is equal to the radius of the innermost Bohr orbit.

IS SAME FOR MANY e^- ATOMS

CHANGES FOR MANY e^- ATOMS

$e^{-r/na_0} \rightarrow e^{-Zr/na_0}$ for hydrogenic
 $Z \rightarrow Z_{\text{eff}}(r)$ in many- e^- atoms

And for a **hydrogenic** atom: atomic no. Z , one electron

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0}$$

$$\psi_{200} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(2 - \frac{Zr}{a_0} \right) e^{-Zr/2a_0}$$

$$\psi_{210} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \cos \theta$$

$$\psi_{21\pm 1} = \frac{1}{8\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \sin \theta e^{\pm i\phi}$$

$$\psi_{300} = \frac{1}{81\sqrt{3\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(27 - 18 \frac{Zr}{a_0} + 2 \frac{Z^2 r^2}{a_0^2} \right) e^{-Zr/3a_0}$$

$$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(6 - \frac{Zr}{a_0} \right) \frac{Zr}{a_0} e^{-Zr/3a_0} \cos \theta$$

$$\psi_{31\pm 1} = \frac{1}{81\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \left(6 - \frac{Zr}{a_0} \right) \frac{Zr}{a_0} e^{-Zr/3a_0} \sin \theta e^{\pm i\phi}$$

$$\psi_{320} = \frac{1}{81\sqrt{6\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} (3 \cos^2 \theta - 1)$$

$$\psi_{32\pm 1} = \frac{1}{81\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} \sin \theta \cos \theta e^{\pm i\phi}$$

$$\psi_{32\pm 2} = \frac{1}{162\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} \sin^2 \theta e^{\pm 2i\phi}$$

$$\mu = \text{reduced mass} = \frac{m_e}{1 + \frac{m_e}{M_{\text{nucleus}}}}$$

$$E_n = -\frac{\mu Z^2 e^4}{2\hbar^2 n^2}, n = 1, 2, 3, \dots$$

$$\langle r_{nl} \rangle = \frac{n^2 a_0}{Z} \left[1 + \frac{1}{2} \left(1 - \frac{\ell(\ell+1)}{n^2} \right) \right]$$

$$= \frac{n^2 a_0}{Z} \left[\begin{array}{l} 1.5 \text{ for } 1s, 1.5 \text{ for } 2s, \\ 1.25 \text{ for } 2p, \dots \text{ converging} \\ \text{to } 1.0 \text{ for very large } n \text{ and } \ell_{\text{max}} = n - 1 \\ \text{(Correspondence Principle limit)} \end{array} \right]$$

$$r_{\text{Bohr}} = \frac{n^2 a_0}{Z}$$

These are written in terms of the parameter

$$a_0 \equiv \frac{\hbar^2}{\mu e^2} = 0.529 \times 10^{-8} \text{ cm}$$

MAKING THE ATOMIC ORBITALS REAL (E.G., FOR CHEMICAL BONDING):

$$\Psi_{nlm_l}(r, \theta, \phi) = R_{nl}(r) \underbrace{\Theta(\theta)}_{\text{REAL}} \underbrace{\frac{1}{\sqrt{2\pi}} e^{im_l \phi}}_{\text{COMPLEX IF } m_l \neq 0}$$

SO JUST TAKE COMB. OF $\pm m_l$ AS:

$$\Psi_{nl(-)}(r, \theta, \phi) = \begin{cases} \frac{1}{2} [\Psi_{nlm_l} + \Psi_{nl-m_l}] \propto R_{nl} \Theta_{lm_l} \cos m_l \phi \\ \frac{1}{2i} [\Psi_{nlm_l} - \Psi_{nl-m_l}] \propto R_{nl} \Theta_{lm_l} \sin m_l \phi \end{cases}$$

REAL

EXAMPLE: 2p ORBITALS

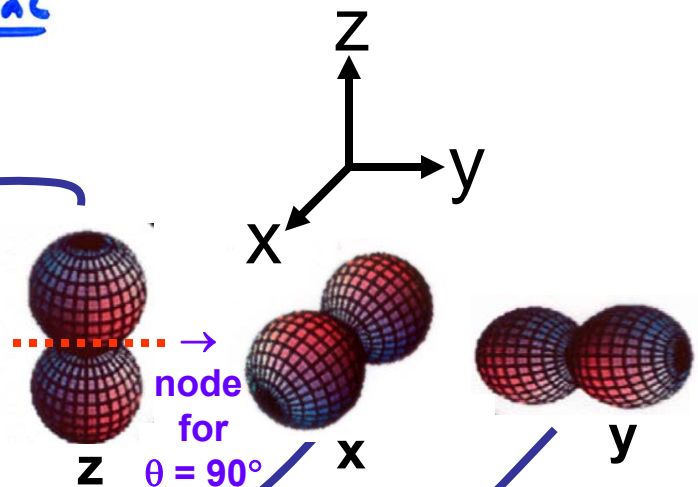
$$\Psi_{210} = \Psi_{2p_0} = \underline{\Psi_{2p_z}} \propto r \cos \theta = z \text{ (ALREADY REAL)}$$

$$\Psi_{211} = \Psi_{2p_{+1}} \propto r \sin \theta e^{i\phi} = r \sin \theta [\cos \phi + i \sin \phi]$$

$$\Psi_{21-1} = \Psi_{2p_{-1}} \propto r \sin \theta e^{-i\phi} = r \sin \theta [\cos \phi - i \sin \phi]$$

$$\frac{1}{2} [\Psi_{2p_{+1}} + \Psi_{2p_{-1}}] = \underline{\Psi_{2p_x}} \propto r \sin \theta \cos \phi = x$$

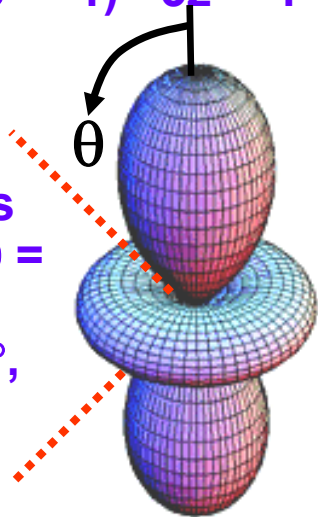
$$\frac{1}{2i} [\Psi_{2p_{+1}} - \Psi_{2p_{-1}}] = \underline{\Psi_{2p_y}} \propto r \sin \theta \sin \phi = y$$



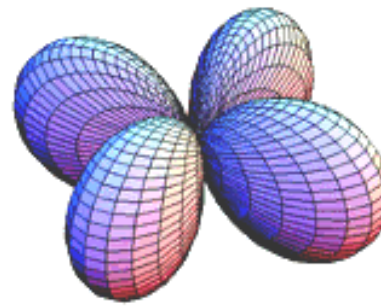
And the same thing for the d orbitals:

$$\propto r^2(3\cos^2\theta - 1) = 3z^2 - r^2$$

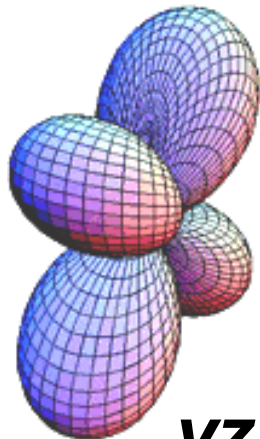
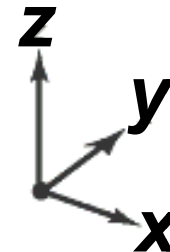
+ nodes
for $\cos\theta =$
 $1/\sqrt{3}$
 $\theta = 54.7^\circ,$
 125.3°



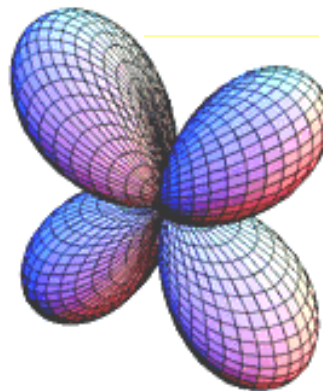
$3z^2 - r^2$



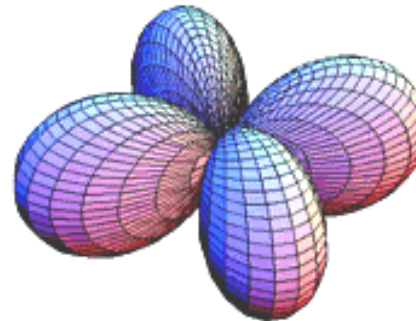
$x^2 - y^2$



yz



zx



xy

The real atomic orbitals, through 3d

$$\psi_{1s} = \frac{1}{\pi^{1/2}} \left(\frac{Z}{a} \right)^{3/2} e^{-Zr/a}$$

$$\psi_{2s} = \frac{1}{4(2\pi)^{1/2}} \left(\frac{Z}{a} \right)^{3/2} \left(2 - \frac{Zr}{a} \right) e^{-Zr/2a}$$

$$\psi_{2p_z} = \frac{1}{4(2\pi)^{1/2}} \left(\frac{Z}{a} \right)^{5/2} r e^{-Zr/2a} \cos \theta$$

$$\psi_{2p_x} = \frac{1}{4(2\pi)^{1/2}} \left(\frac{Z}{a} \right)^{5/2} r e^{-Zr/2a} \sin \theta \cos \varphi$$

$$\psi_{2p_y} = \frac{1}{4(2\pi)^{1/2}} \left(\frac{Z}{a} \right)^{5/2} r e^{-Zr/2a} \sin \theta \sin \varphi$$

$$\psi_{3s} = \frac{1}{81(3\pi)^{1/2}} \left(\frac{Z}{a} \right)^{3/2} \left(27 - 18 \frac{Zr}{a} + 2 \frac{Z^2 r^2}{a^2} \right) e^{-Zr/3a}$$

$$\psi_{3p_z} = \frac{2^{1/2}}{81\pi^{1/2}} \left(\frac{Z}{a} \right)^{5/2} \left(6 - \frac{Zr}{a} \right) r e^{-Zr/3a} \cos \theta$$

$$\psi_{3p_x} = \frac{2^{1/2}}{81\pi^{1/2}} \left(\frac{Z}{a} \right)^{5/2} \left(6 - \frac{Zr}{a} \right) r e^{-Zr/3a} \sin \theta \cos \varphi$$

$$\psi_{3p_y} = \frac{2^{1/2}}{81\pi^{1/2}} \left(\frac{Z}{a} \right)^{5/2} \left(6 - \frac{Zr}{a} \right) r e^{-Zr/3a} \sin \theta \sin \varphi$$

$$\psi_{3d_{z^2}} = \frac{1}{81(6\pi)^{1/2}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} (3 \cos^2 \theta - 1)$$

$$\psi_{3d_{xz}} = \frac{2^{1/2}}{81\pi^{1/2}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin \theta \cos \theta \cos \varphi$$

$$\psi_{3d_{yz}} = \frac{2^{1/2}}{81\pi^{1/2}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin \theta \cos \theta \sin \varphi$$

$$\psi_{3d_{x^2-y^2}} = \frac{1}{81(2\pi)^{1/2}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin^2 \theta \cos 2\varphi$$

$$\psi_{3d_{xy}} = \frac{1}{81(2\pi)^{1/2}} \left(\frac{Z}{a} \right)^{7/2} r^2 e^{-Zr/3a} \sin^2 \theta \sin 2\varphi$$

$$\Psi_{n\ell m_\ell m_s}(r, \theta, \phi, \text{spin}) = \Psi_{n\ell m_\ell}(r, \theta, \phi) \times [\alpha(\uparrow) \text{ or } \beta(\downarrow)]$$

$$Y_{\ell m_\ell}(\theta, \phi) =$$

The atomic orbitals: With spin

"spherical harmonics"

COMPLEX, IF $m \neq 0$ But we can make them real for convenience

TABLE 6.1

NORMALIZED WAVE FUNCTIONS OF THE HYDROGEN ATOM FOR $n = 1, 2,$ AND 3^* ($Z=1 = \text{HYDROGEN}$)

n	ℓ	m_ℓ	$\Phi_{m_\ell}(\phi)$	$\Theta_{\ell m_\ell}(\theta)$	$R_{n\ell}(r)$	$\Psi_{n\ell m_\ell}(r, \theta, \phi) = \Phi_{m_\ell} \Theta_{\ell m_\ell} R_{n\ell}$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2} a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos \theta$	$\frac{1}{2\sqrt{6} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos \theta \rightarrow \text{node for } \theta = 90^\circ$
2	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin \theta$	$\frac{1}{2\sqrt{6} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin \theta e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3} a_0^{3/2}} \left(27 - 18 \frac{r}{a_0} + 2 \frac{r^2}{a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi} a_0^{3/2}} \left(27 - 18 \frac{r}{a_0} + 2 \frac{r^2}{a_0^2}\right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos \theta$	$\frac{4}{81\sqrt{6} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \cos \theta \rightarrow \text{node for } r = 6a_0$
3	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin \theta$	$\frac{4}{81\sqrt{6} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \sin \theta e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4} (3 \cos^2 \theta - 1)$	$\frac{4}{81\sqrt{30} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} (3 \cos^2 \theta - 1) \rightarrow \text{nodes for } \cos^2 \theta = 1/3$
3	2	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{15}}{2} \sin \theta \cos \theta$	$\frac{4}{81\sqrt{30} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin \theta \cos \theta e^{\pm i\phi}$
3	2	± 2	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4} \sin^2 \theta$	$\frac{4}{81\sqrt{30} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi} a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin^2 \theta e^{\pm 2i\phi}$

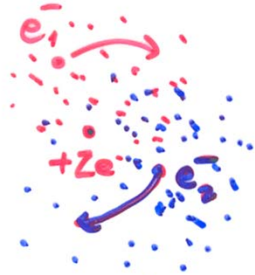
*The quantity $a_0 = 4\pi\epsilon_0\hbar^2/me^2 = 5.3 \times 10^{-11}$ m is equal to the radius of the innermost Bohr orbit.

IS SAME FOR MANY e^- ATOMS

CHANGES FOR MANY e^- ATOMS

$e^{-r/na_0} \rightarrow e^{-Zr/na_0}$ for hydrogenic
 $Z \rightarrow Z_{\text{eff}}(r)$ in many- e^- atoms

What properties do wave functions of overlapping (thus indistinguishable) particles have?—electrons as example:



$\psi = \psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2)$, including spin of both electrons

But labels can't affect any measurable quantity.

E.g. – probability density :

$$|\psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2)|^2 = |\psi(\vec{r}_2, \vec{s}_2; \vec{r}_1, \vec{s}_1)|^2$$

Therefore

$$\begin{aligned} \psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2) &= \pm 1 \psi(\vec{r}_2, \vec{s}_2; \vec{r}_1, \vec{s}_1) \\ &\equiv \hat{P}_{12} \psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2) \end{aligned}$$

with \hat{P}_{12} = permutation operator $\rightarrow \vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2$
and eigenvalues of ± 1

Finally, all particles in two classes :

FERMIONS : (incl. e^- 's) : ψ antisymmetric

$$s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$$

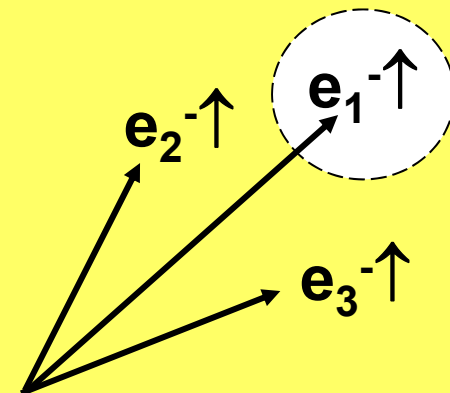
$$\hat{P}_{12} \psi = -1 \psi$$

BOSONS : (incl. photons) : ψ symmetric

$$s = 0, 1, 2, \dots$$

$$\hat{P}_{12} \psi = +1 \psi$$

Probability of finding two electrons at the same point in space with the same spin ($\uparrow\uparrow$ or $\downarrow\downarrow$) is zero: “the Fermi Hole”



→the Exchange Interaction

→Hund's 1st rule & magnetism

The Fermi hole or Exchange/Correlation Hole as seen in theory-silicon atom:

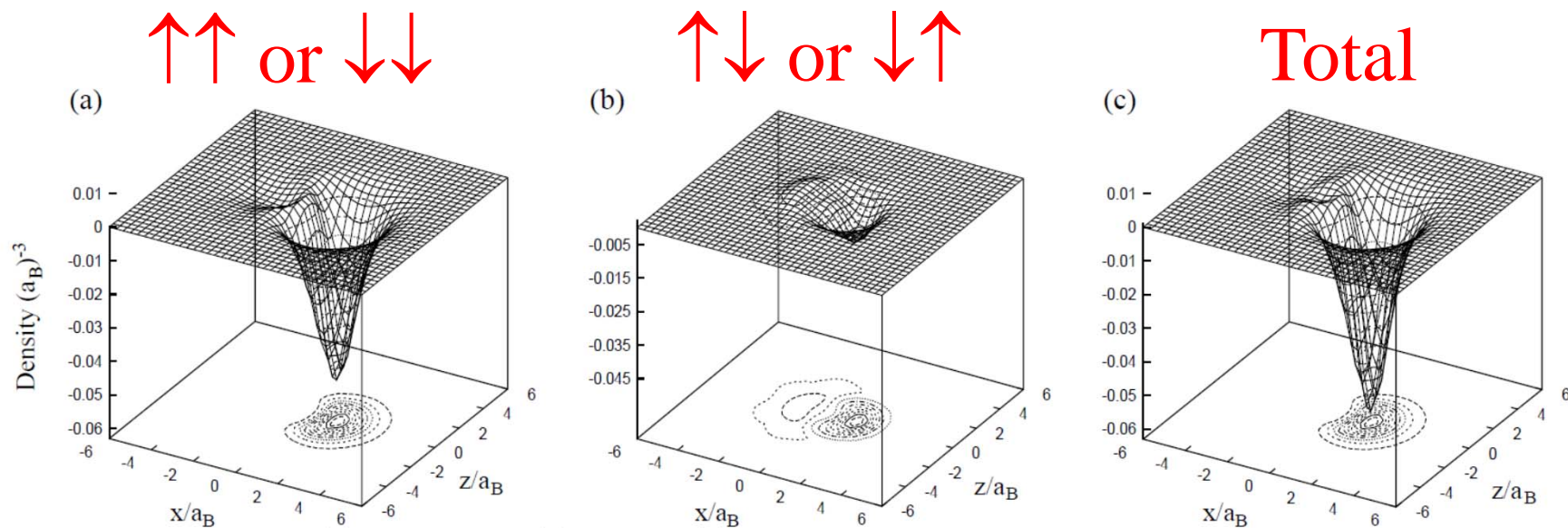


FIG. 1. The same spin (a), opposite spin (b) and total (c) exchange-correlation hole about a spin-up particle located at $1.4 a_B$ from the atom center on the x axis (parallel to the p orbitals) for the ground state of the Si atom in the $L_z = 0$ projection. The surface plot shows the change in density along a plane cutting through the origin along the x and z axes.

Fong (UC Davis) et al., Phys. Rev. A 62, 062507

Antisymmetry and the Pauli Exclusion Principle:

Try Helium, 2 electrons in ground state 1s wave functions, "1s²"

Simple normalized antisymmetric trial wave function is

$$\psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2) = \frac{1}{\sqrt{2}} \left[\varphi_{1s}(\vec{r}_1, \vec{s}_1 = \uparrow) \varphi_{1s}(\vec{r}_2, \vec{s}_2 = \downarrow) - \varphi_{1s}(\vec{r}_1, \vec{s}_1 = \downarrow) \varphi_{1s}(\vec{r}_2, \vec{s}_2 = \uparrow) \right]$$

interchanging labels via permutation operator \hat{P}_{12} gives

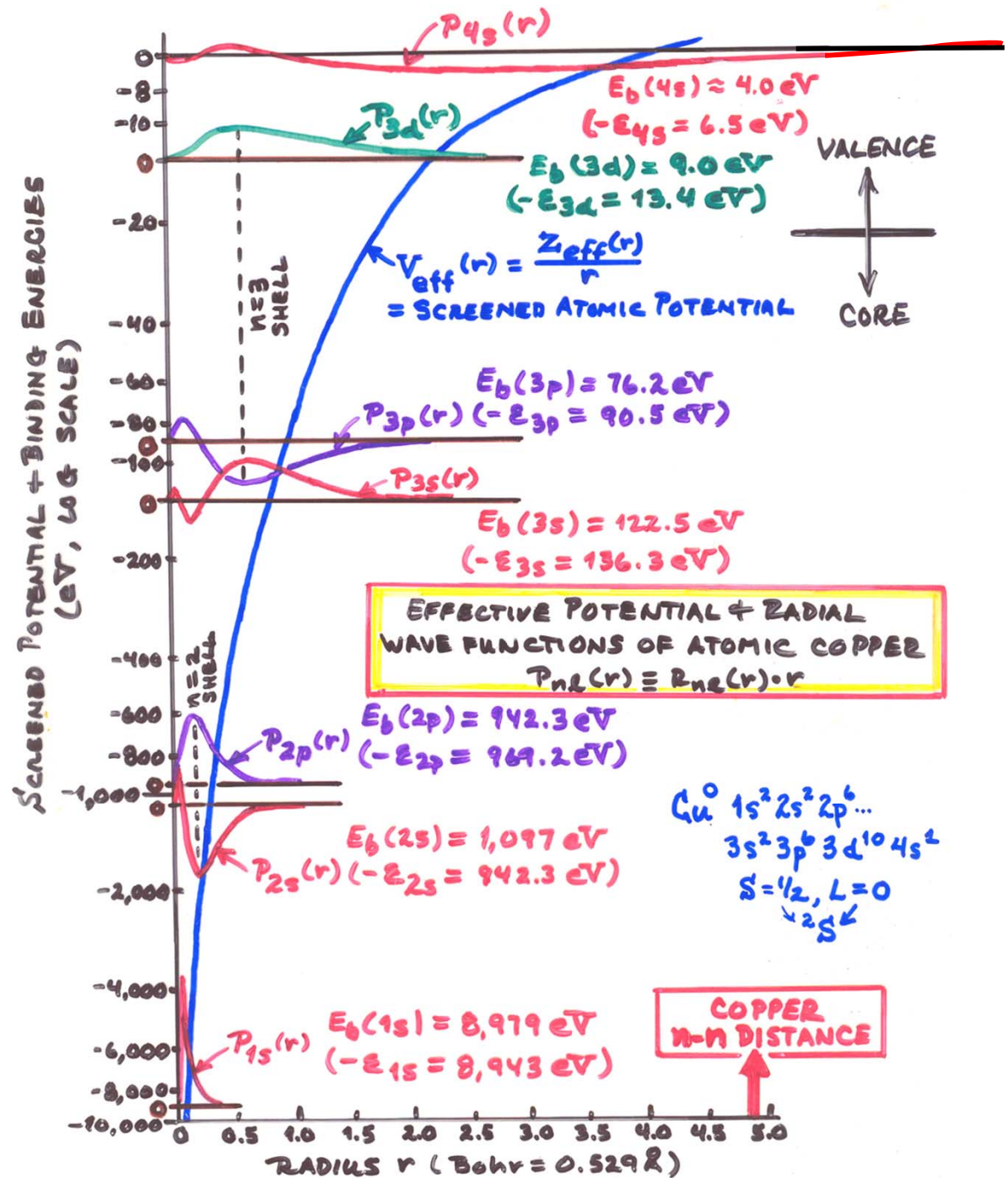
$$\begin{aligned} \hat{P}_{12} \psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2) &\equiv \psi(\vec{r}_2, \vec{s}_2; \vec{r}_1, \vec{s}_1) \\ &= \frac{1}{\sqrt{2}} \left[\varphi_{1s}(\vec{r}_2, \vec{s}_2 = \uparrow) \varphi_{1s}(\vec{r}_1, \vec{s}_1 = \downarrow) - \varphi_{1s}(\vec{r}_2, \vec{s}_2 = \downarrow) \varphi_{1s}(\vec{r}_1, \vec{s}_1 = \uparrow) \right] \\ &= -\psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2), \text{ as required} \end{aligned}$$

Can't tell which electron is spin up--indistinguishable

Also, if we try to put both electrons in 1s with spin-up (\uparrow), first term always cancels second term, and $\psi = 0!$ Therefore, we have the Pauli Exclusion Principle !!! \rightarrow

Intraatomic electron screening in many-electron atoms--a self-consistent Q.M. calculation

Plus radial one-electron functions:
 $P_{nl}(r) \equiv rR_{nl}(r)$



OBSERVED (+ CALCULATED) ORDER OF FILLING ATOMIC LEVELS:

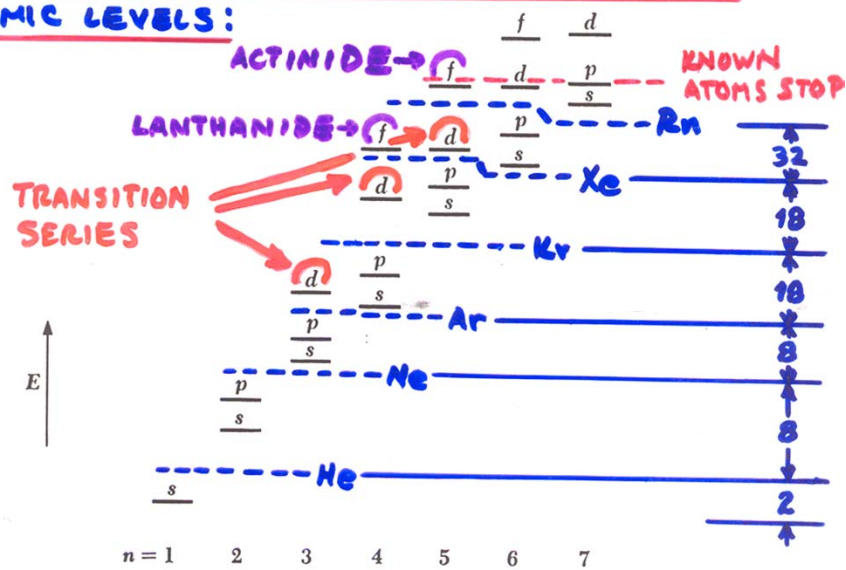
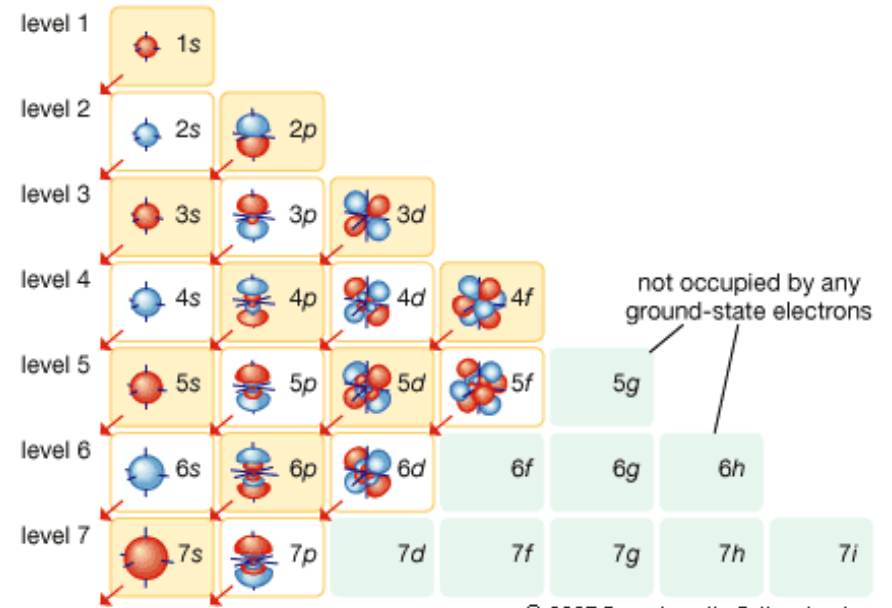


FIGURE 7.13 The sequence of quantum states in an atom. Not to scale.



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TRANSITION METALS

Periodic Table, with the Outer Electron Configurations of Neutral Atoms in Their Ground States

The notation used to describe the electronic configuration of atoms and ions is discussed in all textbooks of introductory atomic physics. The letters s, p, d, \dots signify electrons having orbital angular momentum $0, 1, 2, \dots$ in units \hbar ; the number to the left of the letter denotes the principal quantum number of one orbit, and the superscript to the right denotes the number of electrons in the orbit.

H ¹																	He ²				
1s																	1s ²				
Li ³	Be ⁴															B ⁵	C ⁶	N ⁷	O ⁸	F ⁹	Ne ¹⁰
2s	2s ²															2s ² 2p	2s ² 2p ²	2s ² 2p ³	2s ² 2p ⁴	2s ² 2p ⁵	2s ² 2p ⁶
Na ¹¹	Mg ¹²															Al ¹³	Si ¹⁴	P ¹⁵	S ¹⁶	Cl ¹⁷	Ar ¹⁸
3s	3s ²															3s ² 3p	3s ² 3p ²	3s ² 3p ³	3s ² 3p ⁴	3s ² 3p ⁵	3s ² 3p ⁶
K ¹⁹	Ca ²⁰	Sc ²¹	Ti ²²	V ²³	Cr ²⁴	Mn ²⁵	Fe ²⁶	Co ²⁷	Ni ²⁸	Cu ²⁹	Zn ³⁰	Ga ³¹	Ge ³²	As ³³	Se ³⁴	Br ³⁵	Kr ³⁶				
4s	4s ²	4s ²	4s ²	4s ²	4s	4s ²	4s ²	4s ²	4s ²	4s	4s ²	4s ² 4p	4s ² 4p ²	4s ² 4p ³	4s ² 4p ⁴	4s ² 4p ⁵	4s ² 4p ⁶				
Rb ³⁷	Sr ³⁸	Y ³⁹	Zr ⁴⁰	Nb ⁴¹	Mo ⁴²	Tc ⁴³	Ru ⁴⁴	Rh ⁴⁵	Pd ⁴⁶	Ag ⁴⁷	Cd ⁴⁸	In ⁴⁹	Sn ⁵⁰	Sb ⁵¹	Te ⁵²	I ⁵³	Xe ⁵⁴				
5s	5s ²	5s ²	5s ²	5s	5s	5s	5s	5s	-	5s	5s ²	5s ² 5p	5s ² 5p ²	5s ² 5p ³	5s ² 5p ⁴	5s ² 5p ⁵	5s ² 5p ⁶				
Cs ⁵⁵	Ba ⁵⁶	La ⁵⁷	Hf ⁷²	Ta ⁷³	W ⁷⁴	Re ⁷⁵	Os ⁷⁶	Ir ⁷⁷	Pt ⁷⁸	Au ⁷⁹	Hg ⁸⁰	Tl ⁸¹	Pb ⁸²	Bi ⁸³	Po ⁸⁴	At ⁸⁵	Rn ⁸⁶				
6s	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	-	6s	6s	6s ²	6s ² 6p	6s ² 6p ²	6s ² 6p ³	6s ² 6p ⁴	6s ² 6p ⁵	6s ² 6p ⁶				
Fr ⁸⁷	Ra ⁸⁸	Ac ⁸⁹																			
7s	7s ²	7s ²																			
			4f ¹ ... 4f(5d) FILLING ... 4f ¹⁴																		
			Ce ⁵⁸	Pr ⁵⁹	Nd ⁶⁰	Pm ⁶¹	Sm ⁶²	Eu ⁶³	Gd ⁶⁴	Tb ⁶⁵	Dy ⁶⁶	Ho ⁶⁷	Er ⁶⁸	Tm ⁶⁹	Yb ⁷⁰	Lu ⁷¹					
			4f ²	4f ³	4f ⁴	4f ⁵	4f ⁶	4f ⁷	4f ⁷	4f ⁸	4f ¹⁰	4f ¹¹	4f ¹²	4f ¹³	4f ¹⁴	5d					
			6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²	6s ²					
			5f ¹ FILLING ...																		
			Th ⁹⁰	Pa ⁹¹	U ⁹²	Np ⁹³	Pu ⁹⁴	Am ⁹⁵	Cm ⁹⁶	Bk ⁹⁷	Cf ⁹⁸	Es ⁹⁹	Fm ¹⁰⁰	Md ¹⁰¹	No ¹⁰²	Lr ¹⁰³					
			-	5f ²	5f ³	5f ⁵	5f ⁶	5f ⁷	5f ⁷												
			6d ²	6d	6d				6d												
			7s ²	7s ²	7s ²	7s ²	7s ²	7s ²	7s ²												

p¹ p² p³ p⁴ p⁵ p⁶

d¹ d² d³ d⁴ d⁵ d⁶ d⁷ d⁸ d⁹ d¹⁰

☐ = EXCEPTIONS
 ◻ = EXCEPTIONS

→ d⁵ + d¹⁰ : 1/2 FILLED / FILLED MORE STABLE

RARE EARTHS

ACTINIDES

OBSERVED (+ CALCULATED) ORDER OF FILLING ATOMIC LEVELS:

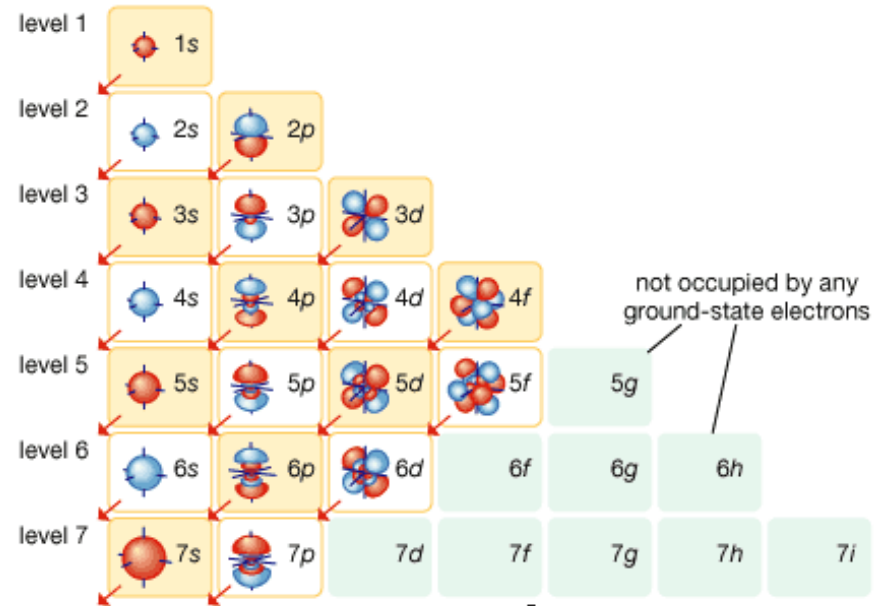
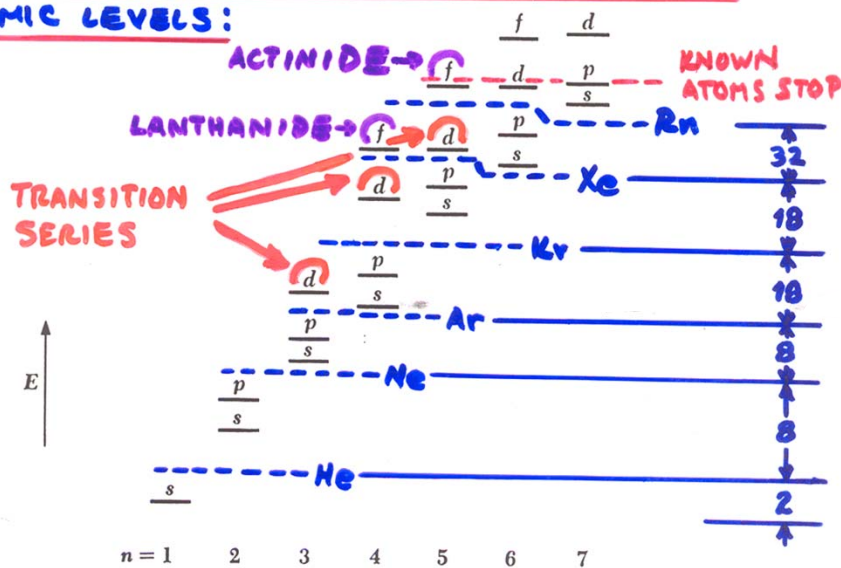


FIGURE 7.13 The sequence of quantum states in an atom. Not to scale.

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EXAMPLE CONFIGURATIONS:

Z	ATOM	CONFIG.	GROUND-STATE OPEN SHELL COUPLING?
8	O	$1s^2 2s^2 2p^4$	$\uparrow\downarrow$ \uparrow \uparrow $2p_{-1}$ $2p_0$ $2p_{+1}$
26	Fe	$1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$	$\uparrow\downarrow$ \uparrow \uparrow \uparrow \uparrow $3d_{-2}$ $3d_{-1}$ $3d_0$ $3d_{+1}$ $3d_{+2}$ ⇒ LARGE μ_{3d} + MAGNETISM
63	Eu	$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^7 6s^2$	\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow $4f_{-3}$ $4f_{-2}$ $4f_{-1}$ $4f_0$ $4f_{+1}$ $4f_{+2}$ $4f_{+3}$ ALSO MAGNETIC!

Exchange interaction.
Hund's First Rule:
highest total spin angular momentum

The quantum mechanics of covalent bonding in molecules: H_2^+ with one electron

$$\begin{aligned}\varphi_- &= \varphi_{\text{antibonding}} \\ &\cong \varphi_{1sa} - \varphi_{1sb}\end{aligned}$$

$$\begin{aligned}\varphi_+ &= \varphi_{\text{bonding}} \\ &\cong \varphi_{1sa} + \varphi_{1sb}\end{aligned}$$

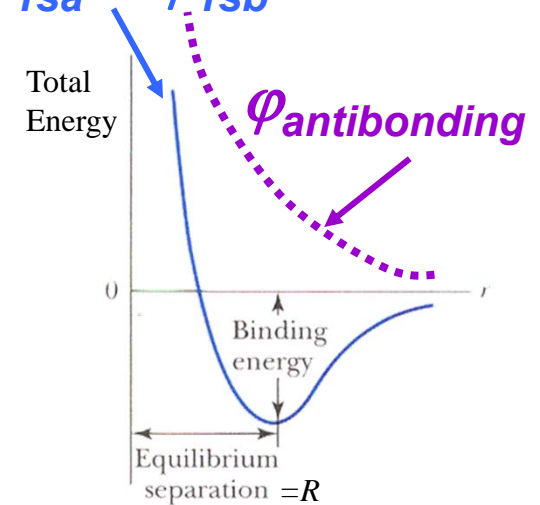


FIGURE 10.2 The net potential energy curve, showing the equilibrium separation and binding energy.

Bonding in H_2^+ :
a linear combination
of atomic orbitals
(LCAO) or tight-
binding (TB) picture

The bonding state

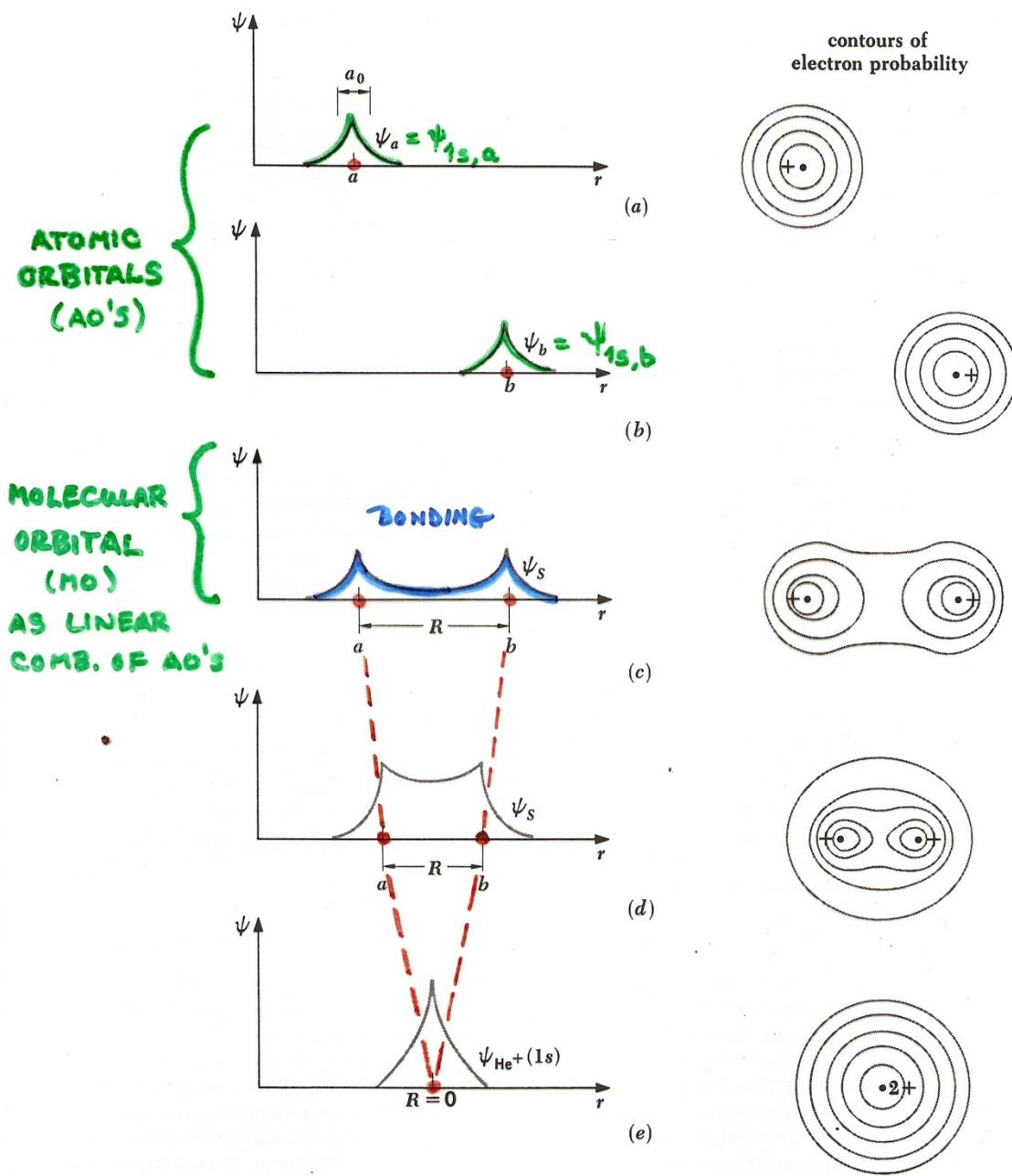


FIGURE 8.5 The combination of two hydrogen-atom 1s wave functions to form the symmetric H_2^+ wave function ψ_s .

**Bonding in H_2^+ :
a linear combination
of atomic orbitals
(LCAO) or tight-
binding (TB) picture**

The anti-bonding state

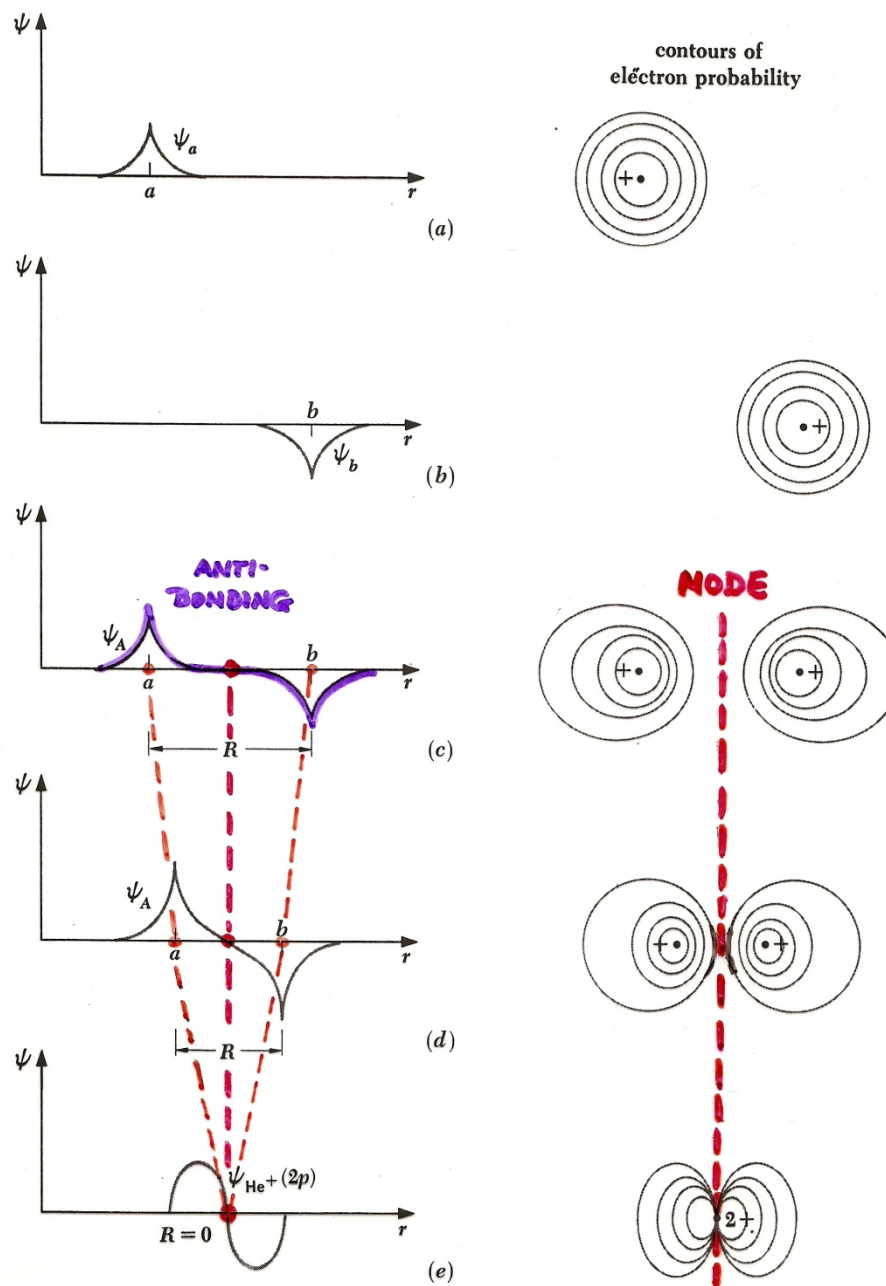


FIGURE 8.6 The combination of two hydrogen-atom 1s wave functions to form the antisymmetric H_2^+ wave function ψ_A .

Bonding in H_2^+ :
a linear combination
of atomic orbitals
(LCAO) or tight-
binding (TB) picture

The energies involved

$$V_p = \frac{e^2}{4\pi\epsilon_0 R} = \frac{14.4}{R(\text{\AA})} (\text{eV})$$

- V_p = Proton potential energy
- E_S = Electron energy (symmetric state)
- E_S^{total} = H_2^+ energy (symmetric state) - **BONDING**
- E_A = Electron energy (antisymmetric state)
- E_A^{total} = H_2^+ energy (antisymmetric state) - **ANTI-BONDING**

$$E(\text{He}^+ 2p) = -\frac{(2)^2 e^2}{8\pi\epsilon_0 a_0 (2)^2} = -13.6 \text{ eV}$$



$$E(\text{He}^+ 1s) = -\frac{(2)^2 e^2}{8\pi\epsilon_0 a_0 (1)^2} = -54.4 \text{ eV}$$



$$E(H1s) = -\frac{(1)^2 e^2}{8\pi\epsilon_0 a_0 (1)^2} = -13.6 \text{ eV}$$

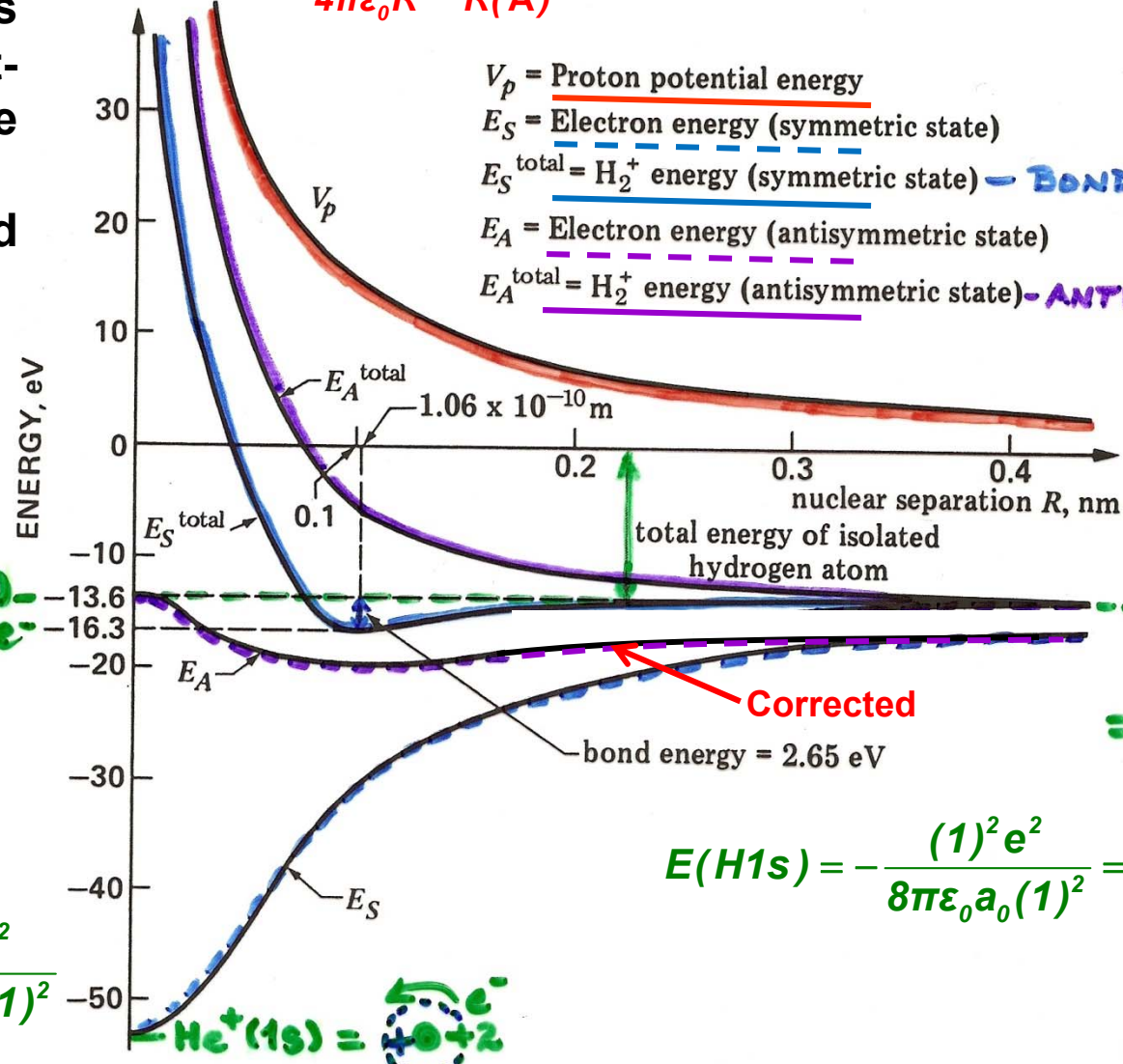


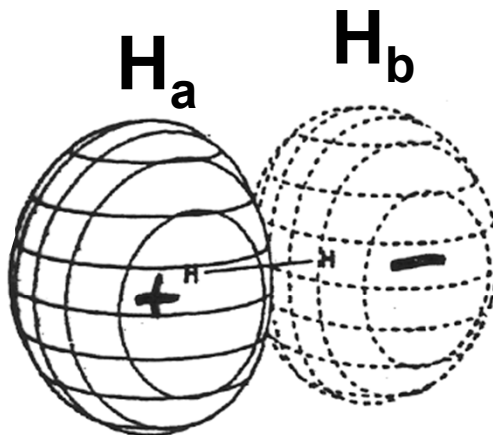
FIGURE 8.7 Electron, proton repulsion, and total energies in H_2^+ as a function of nuclear separation R for the symmetric and antisymmetric states. The antisymmetric state has no minimum in its total energy.

2 electrons:
The linear
combination of
atomic orbitals
(LCAO-MO)
or tight-
binding
picture for H₂:

III. MOLECULAR ORBITAL DRAWINGS

1. Hydrogen

Symmetry: D_{∞h}

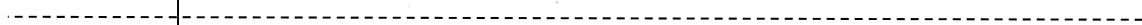


Anti-Bonding

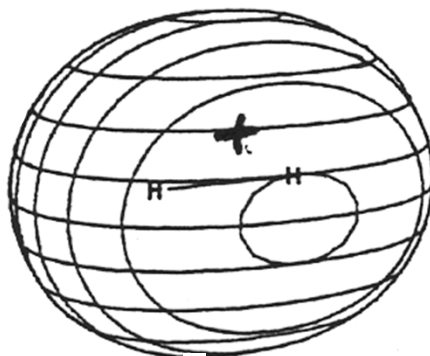
$$\varphi_{anti}^{MO} \cong \varphi_{1sa} - \varphi_{1sb}$$

$$1\sigma_u \quad \epsilon = 0.2656 \text{ a.u.} = +7.21 \text{ eV}$$

ϵ positive
(unoccupied)



ϵ negative
(occupied)



Bonding

$$\varphi_{bonding}^{MO} \cong \varphi_{1sa} + \varphi_{1sb}$$

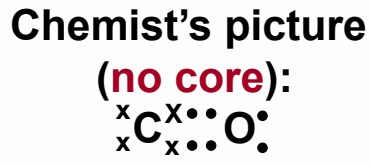
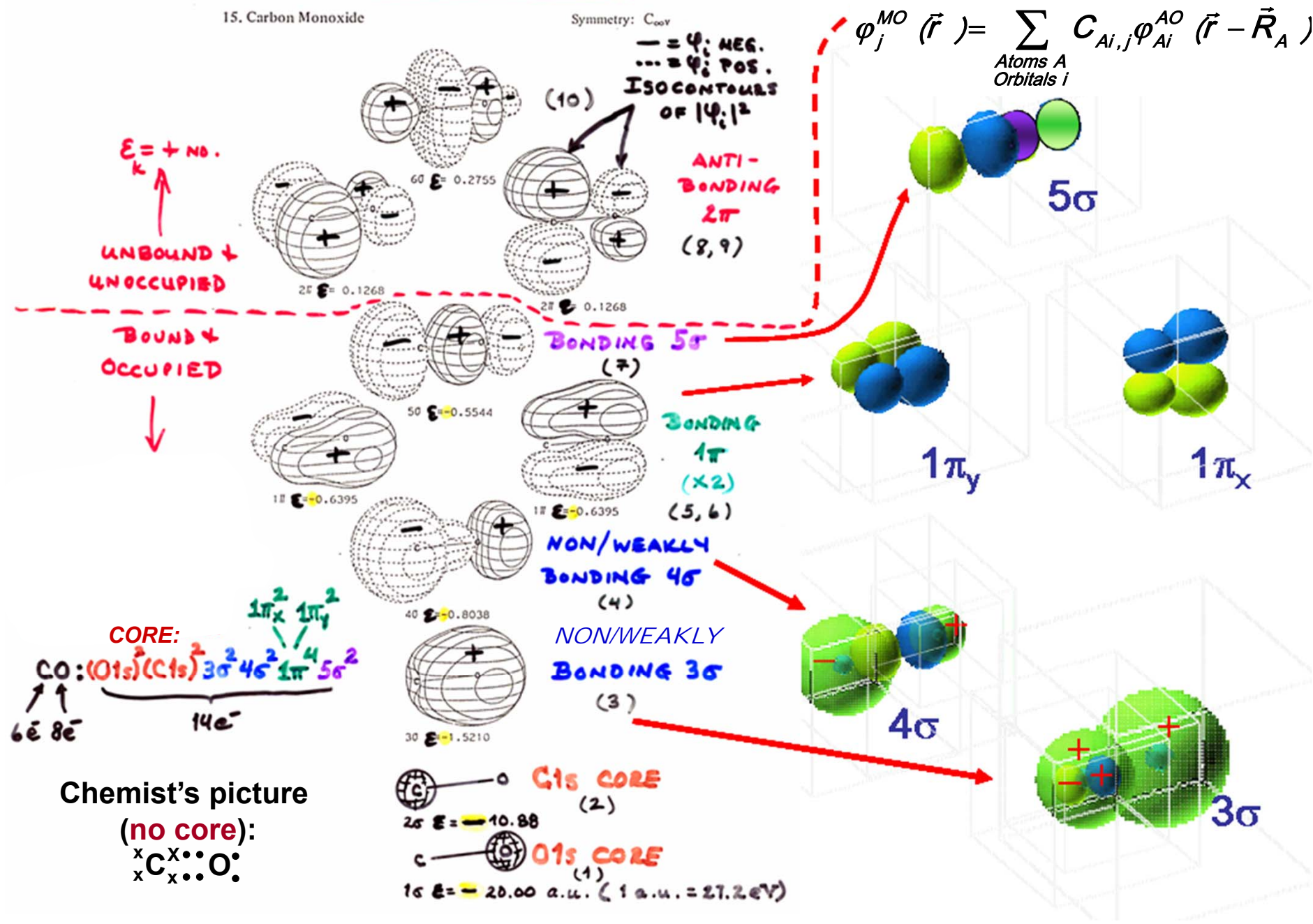
$$1\sigma_g \quad \epsilon = -0.5944 \text{ a.u.} = -16.16 \text{ eV}$$

(Compare - 13.61 for H atom 1s)

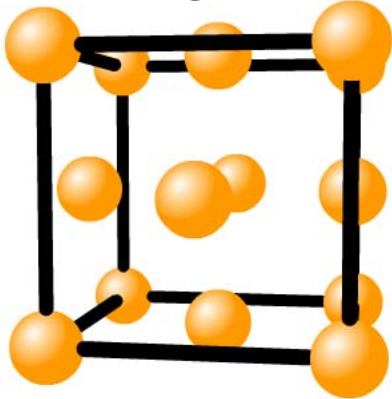
The LCAO-MO or tight-binding picture for CO:

Atomic orbital makeup

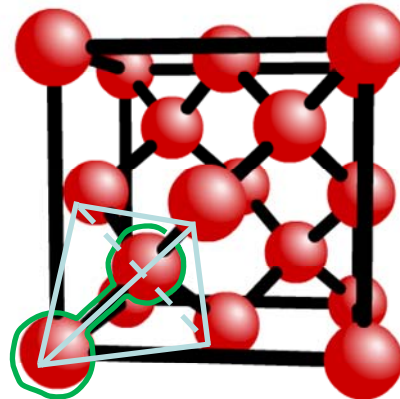
$$\varphi_j^{MO}(\vec{r}) = \sum_{\substack{\text{Atoms } A \\ \text{Orbitals } i}} C_{Ai,j} \varphi_{Ai}^{AO}(\vec{r} - \vec{R}_A)$$



Face-centered cubic
(e.g. Cu)



Diamond
(e.g., C, Si, Ge)



Basis

Ordered packing of atoms in solids

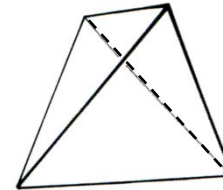
[Good websites/downloads](http://www.dawgSDK.org/crystal/en/library/fcc#0002)

for simple structures:

<http://www.dawgSDK.org/crystal/en/library/fcc#0002>

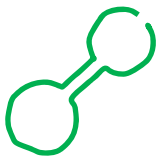
<http://demonstrations.wolfram.com/CrystalViewer/>

Tetrahedron



And another website for various structures, orbitals, etc:

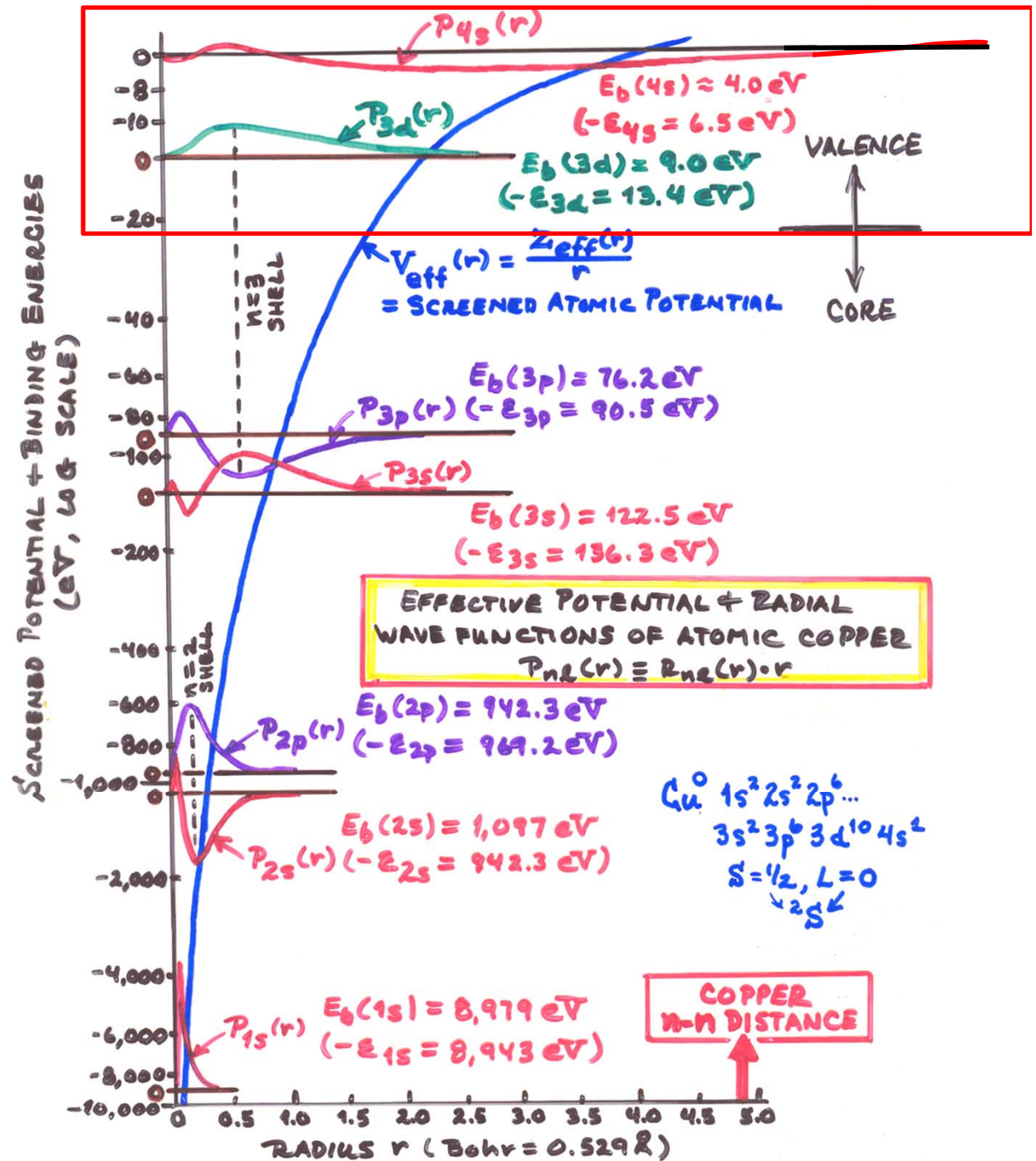
[http://www.chemtube3d.com/solidstate/simplecubic\(final\).htm](http://www.chemtube3d.com/solidstate/simplecubic(final).htm)



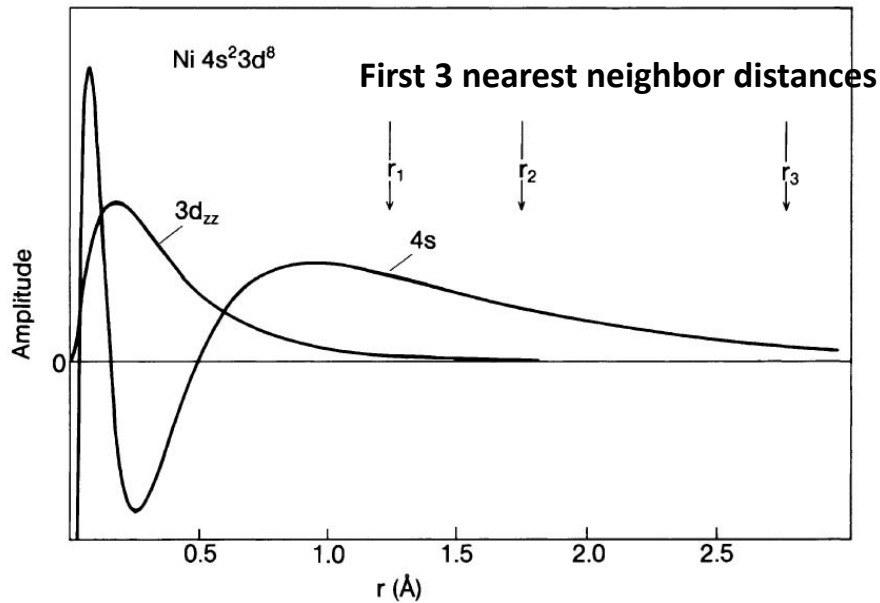
Basis

Intraatomic electron screening in many-electron atoms--a self-consistent Q.M. calculation

Plus radial one-electron functions:
 $P_{nl}(r) \equiv rR_{nl}(r)$



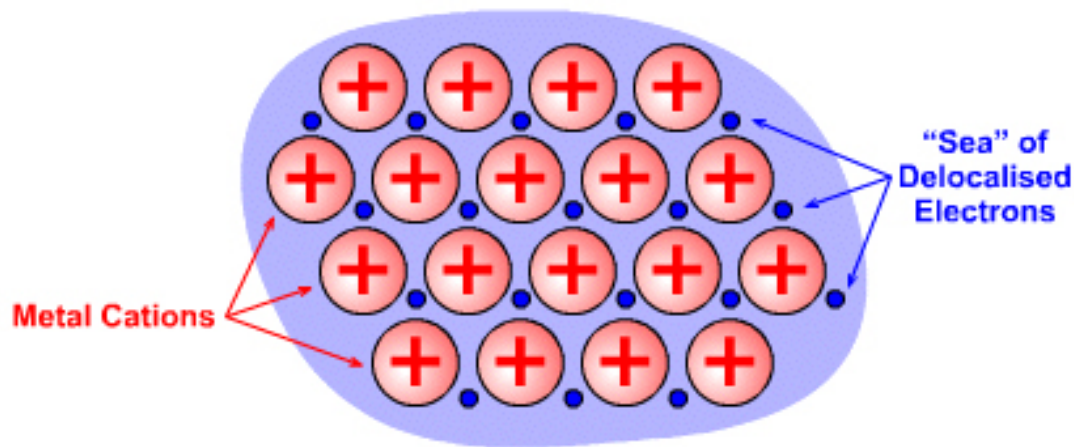
Metallic bonding, e.g. Ni and Cu, with 4s electrons highly delocalized and mobile



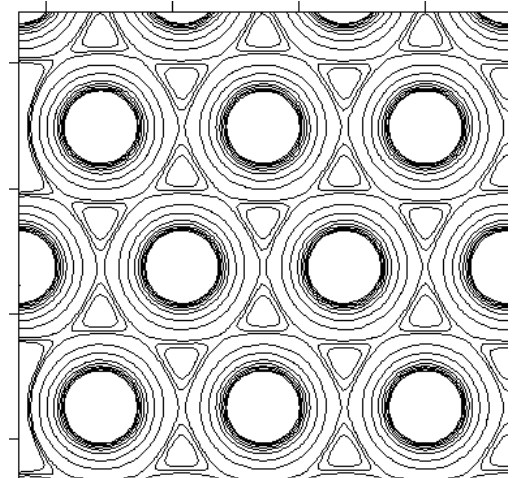
$$\psi_{4s} \approx C \exp(ik \cdot r)$$

$$E_{4s} \approx \hbar^2 k^2 / 2m_e$$

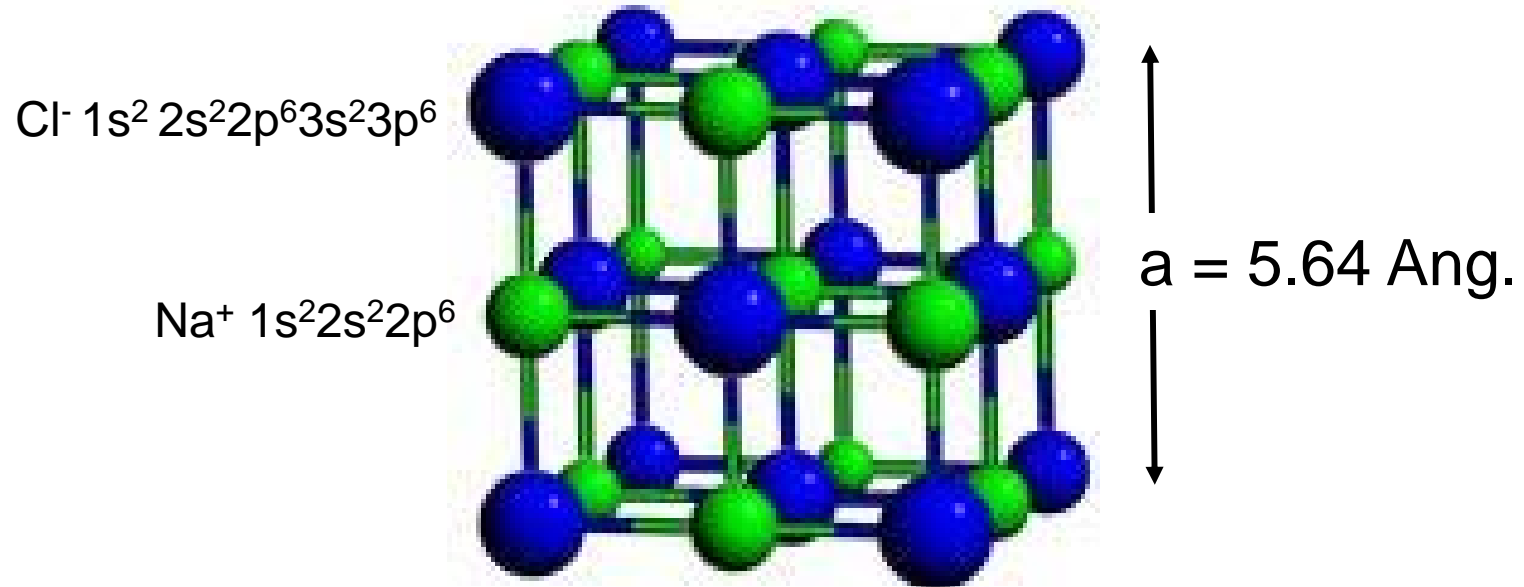
Fig. 1.9. The amplitude of the $3d_{zz}$ -wavefunction and the $4s$ -wavefunction of Ni [1.4]. The half-distances to the first, second and third nearest neighbors (r_1, r_2 and r_3) are shown for comparison



Electron density in copper



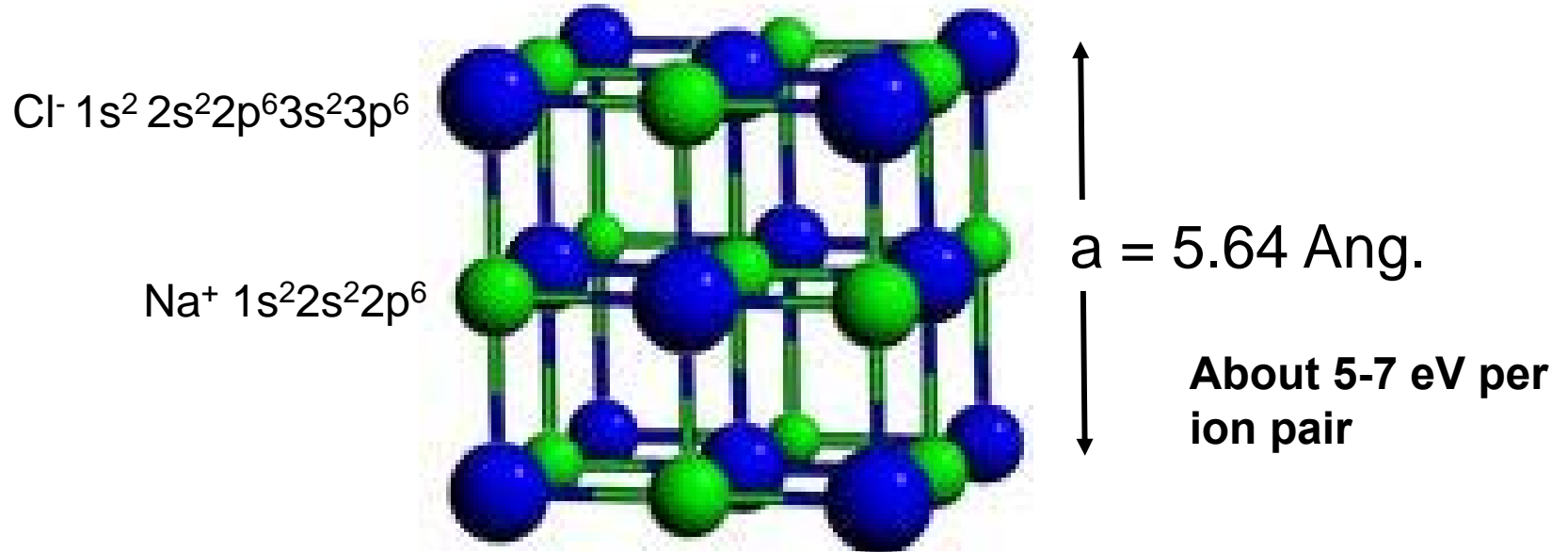
Ionic solids—another limit: e.g NaCl



Nobel gas configuration ions, with long-range point-charge
Coulomb attraction or repulsion, plus
short-range nearest-neighbor repulsion:

$$V = V_{Coul} + V_{Rep} = \sum_{i=1}^{\infty} \frac{(+ \text{ or } -)n_i q_i e^2}{4\pi\epsilon_n r_i} + \lambda e^{-\frac{r_i}{\rho}} \left[\text{or } B/r^n, n \approx 6, \text{ Ibach and Luth} \right]$$

Ionic solids—another limit: e.g NaCl



Nobel gas configuration ions, with long-range point-charge Coulomb attraction or repulsion, plus short-range nearest-neighbor repulsion:

$$V = V_{Coul} + V_{Rep} = \sum_{i=1}^{\infty} \frac{(+ \text{ or } -)n_i q_i e^2}{4\pi\epsilon_0 r_i} + \lambda e^{-\frac{r_i}{\rho}} \left[\text{or } B/r^n, n \approx 6, \text{ Ibach and Luth} \right]$$

as e.g., for NaCl: $\lambda = 1.05 \times 10^{-8} \text{ erg}$, $\rho = 0.321 \text{ Angstroms}$

$$V = \frac{e^2}{4\pi\epsilon_0} \left[-\frac{6}{a/2} + \frac{12}{\sqrt{2}a/2} - \frac{8}{\sqrt{3}a/2} \dots \right] + 1.05e^{-\frac{(a/2)(\text{Ang.})}{0.321}}$$

$$= \frac{e^2}{2\pi\epsilon_0 a} \left[-6 + \frac{12}{\sqrt{2}} - \frac{8}{\sqrt{3}} \dots \right] + 1.05e^{-\frac{(a/2)(\text{Ang.})}{0.321}} = -\frac{e^2}{2\pi\epsilon_0 a} [1.7476] + 1.05e^{-\frac{(a/2)(\text{Ang.})}{0.321}}$$

= α = Madelung constant

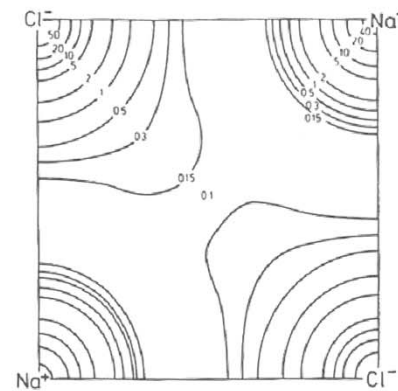
Electronegativities of some elements

Table 1.2. The electronegativity of selected elements [1.1]

H							
2.1							
Li	Be	B	C	N	O	F	
1.0	1.5	2.0	2.5	3.0	3.5	4.0	MOST
Na	Mg	Al	Si	P	S	Cl	
0.9	1.2	1.5	1.8	2.1	2.5	3.0	
K	Ca	Sc	Ge	As	Se	Br	
0.8	1.0	1.3	1.8	2.0	2.4	2.8	
Rb	Sr	Y	Sn	Sb	Te	I	
0.8	1.0	1.3	1.8	1.9	2.1	2.5	

Charge distributions in ionic and covalent bonding

IONIC :



COVALENT :



Fig. 1.8. Density of valence electrons in a typical ionic crystal (NaCl) and in a typical covalently bound crystal (Si) [1.2, 3]. One clearly sees the concentration of charge along the bond between Si atoms, whereas in the ionic bonding, the electrons are almost spherically distributed around the ions

LOSS OF
 e^- CHARGE
RELATIVE
TO ATOMIC
DENSITIES

GAIN OF
 e^- CHARGE
RELATIVE
TO ATOMIC
DENSITIES

COVALENT:

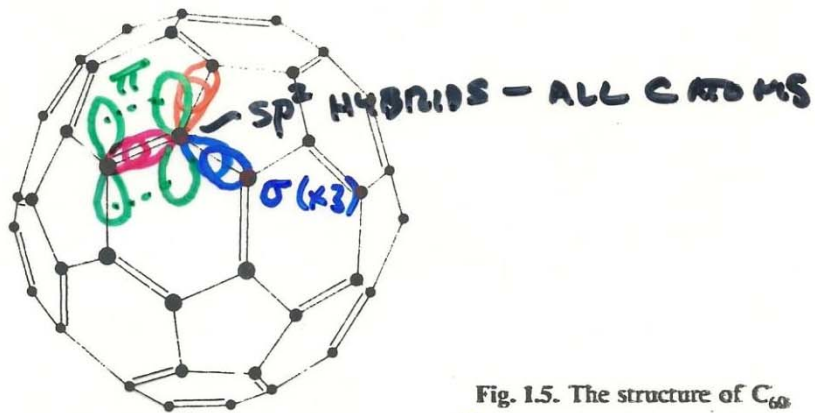
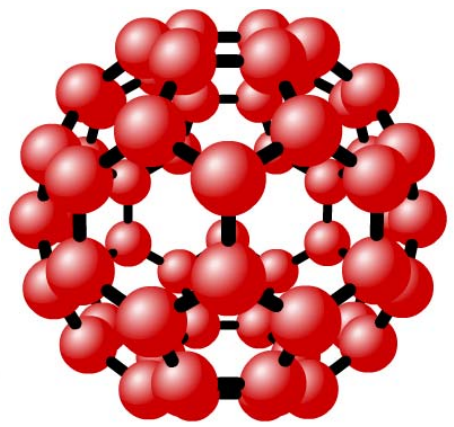
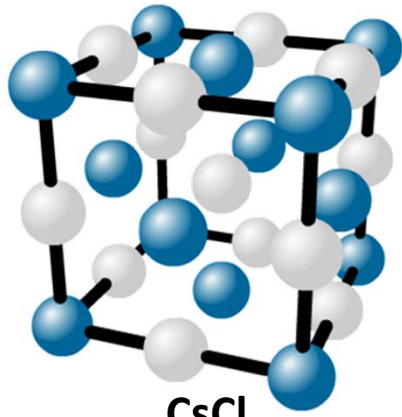


Fig. 1.5. The structure of C₆₀
"buckyball"
⇓
"fullerenes"

Buckyball



NaCl



CsCl

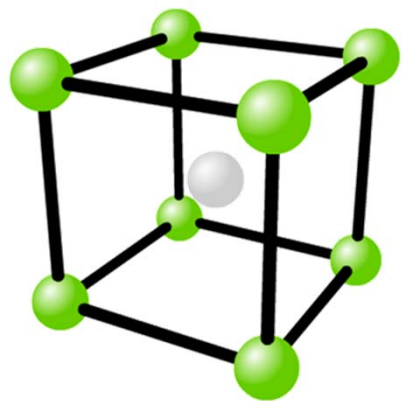
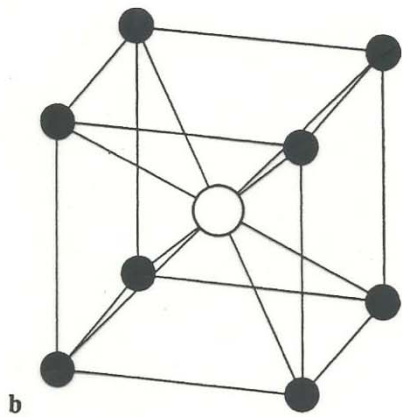
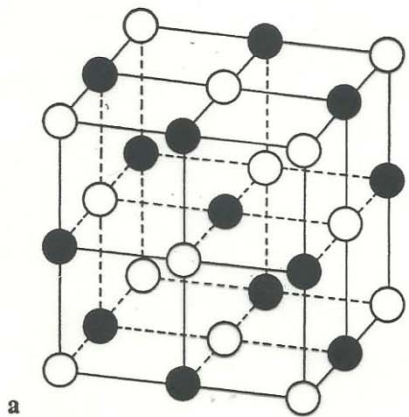


Fig. 1.6. The two structures typical ionic bonding in solids: a NaCl struct b CsCl structure

IONIC:



Covalent and Van der Waals Bonding in Graphite

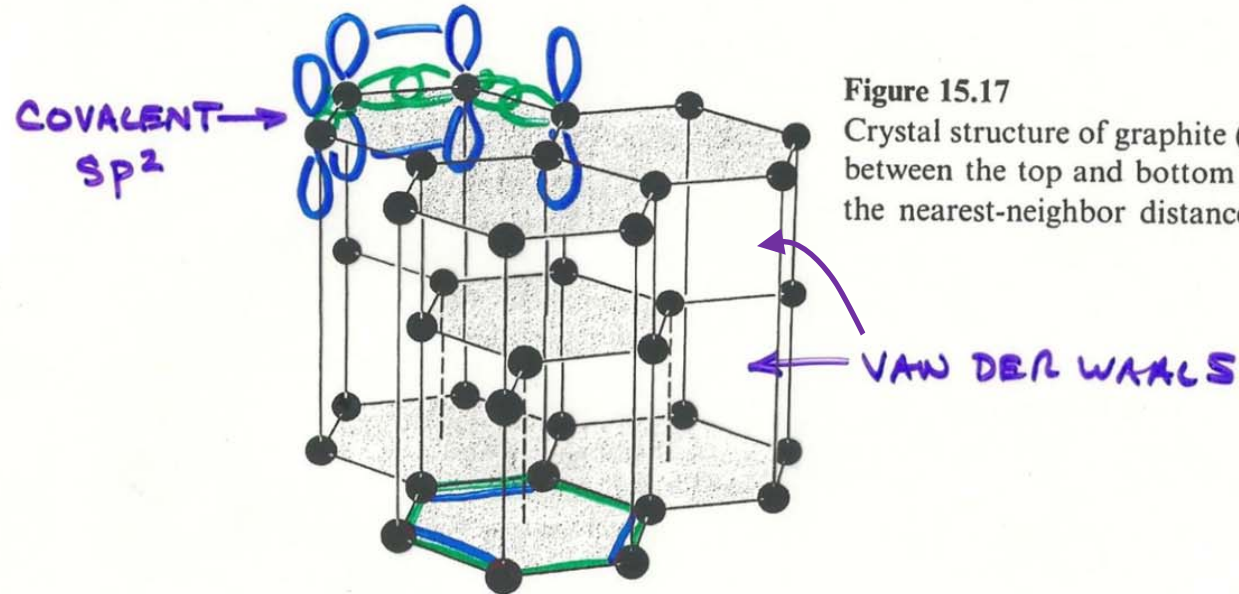


Figure 15.17

Crystal structure of graphite (not to scale). The distance between the top and bottom planes is almost 4.8 times the nearest-neighbor distance within planes.

Covalent and Hydrogen Bonding in Ice

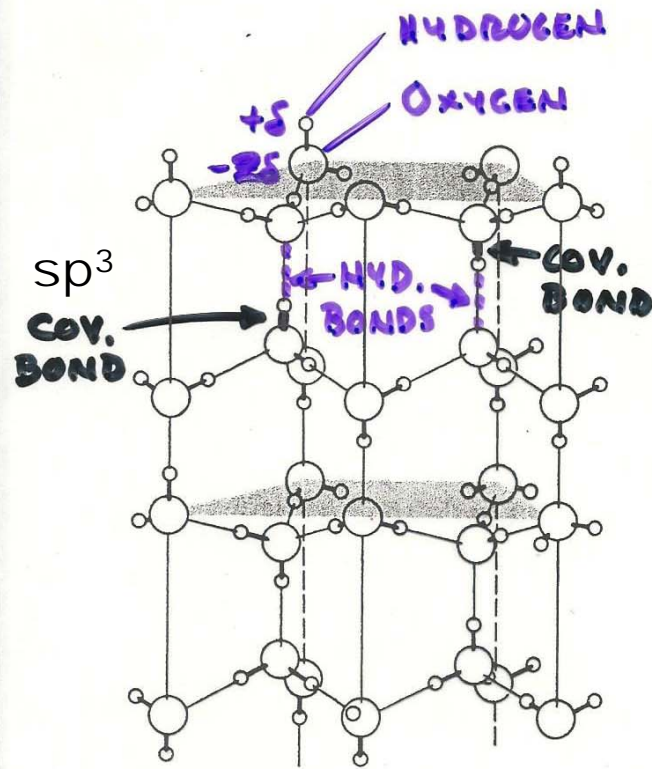
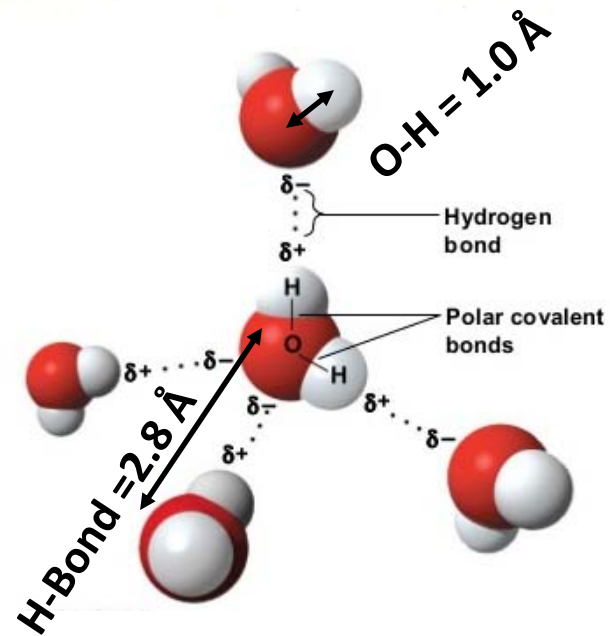
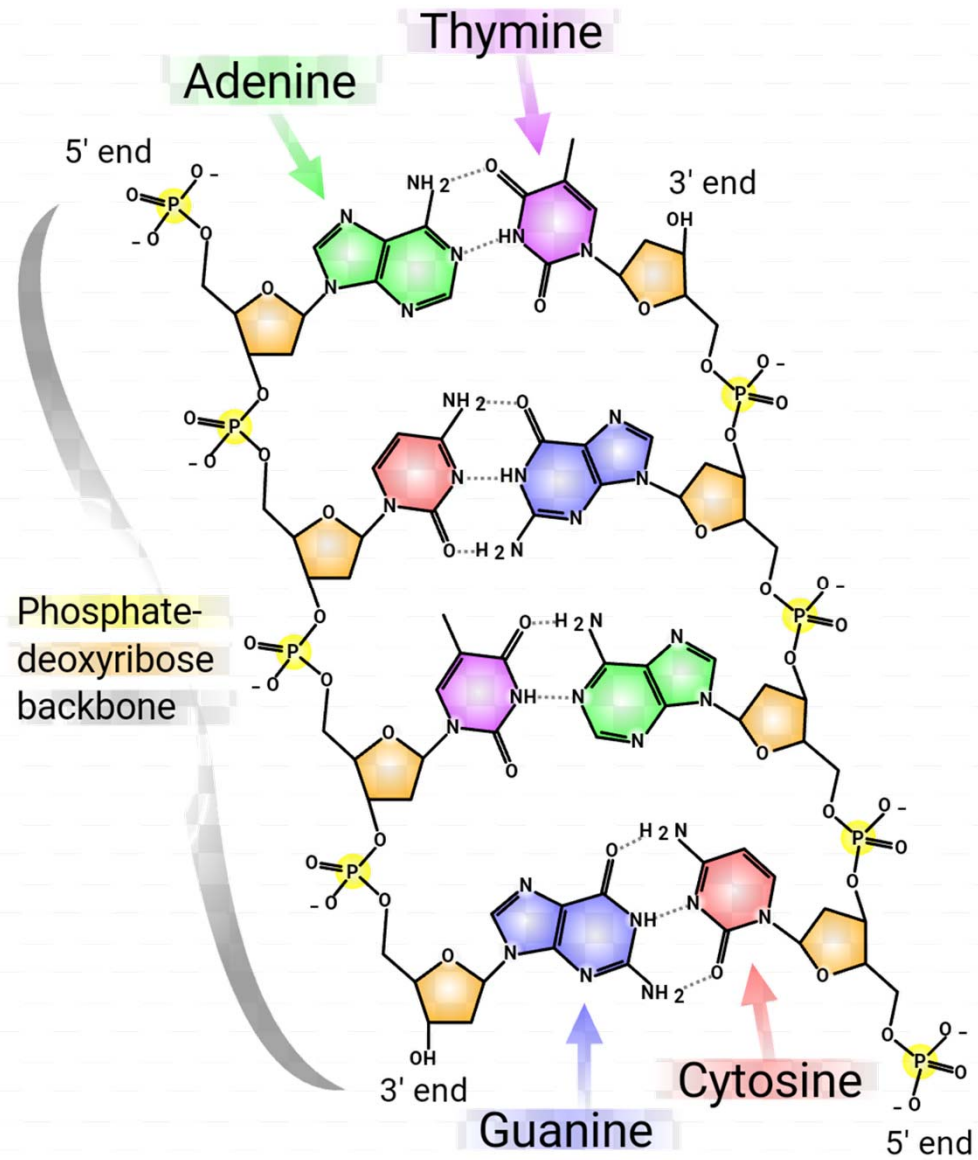


Figure 19.11

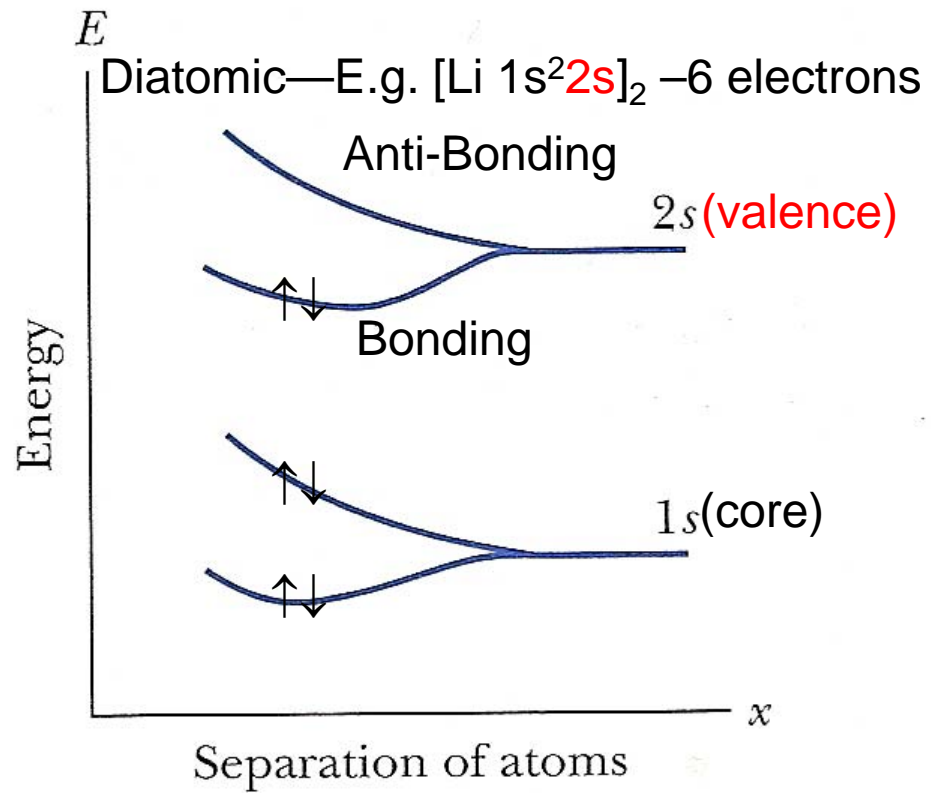
The crystal structure of one of the many phases of ice. The large circles are oxygen ions; the small circles are protons. Ice is an example in which hydrogen bonding plays a crucial role. (After L. Pauling, *The Nature of The Chemical Bond*, 3rd. ed., Cornell University Press, Ithaca, New York, 1960.)



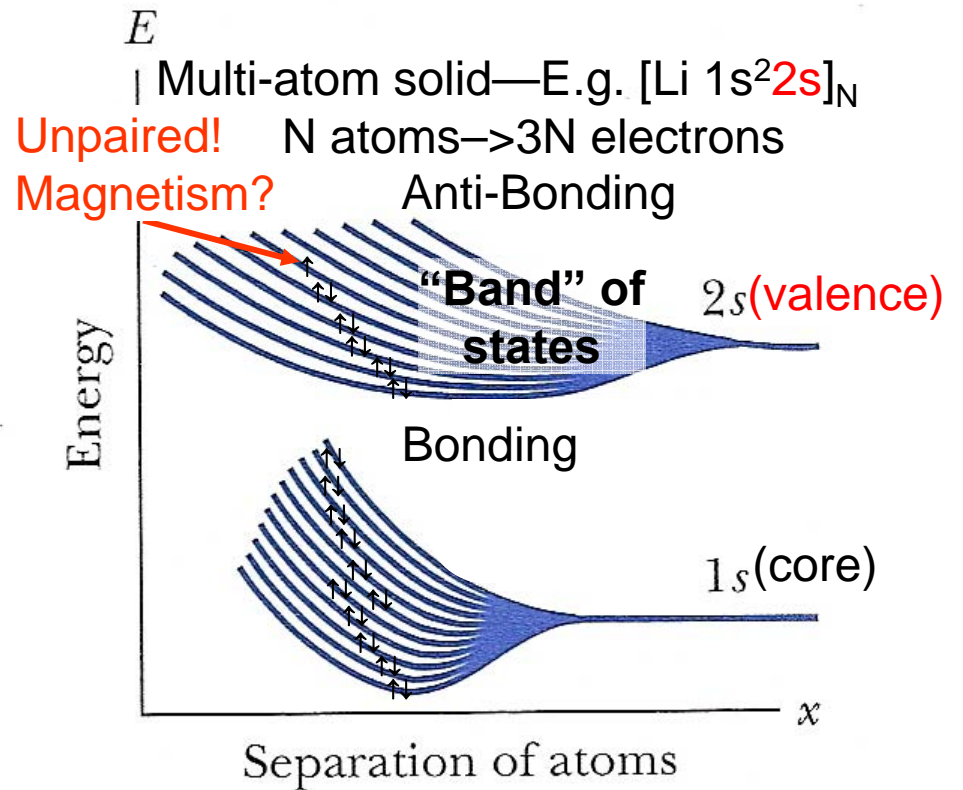
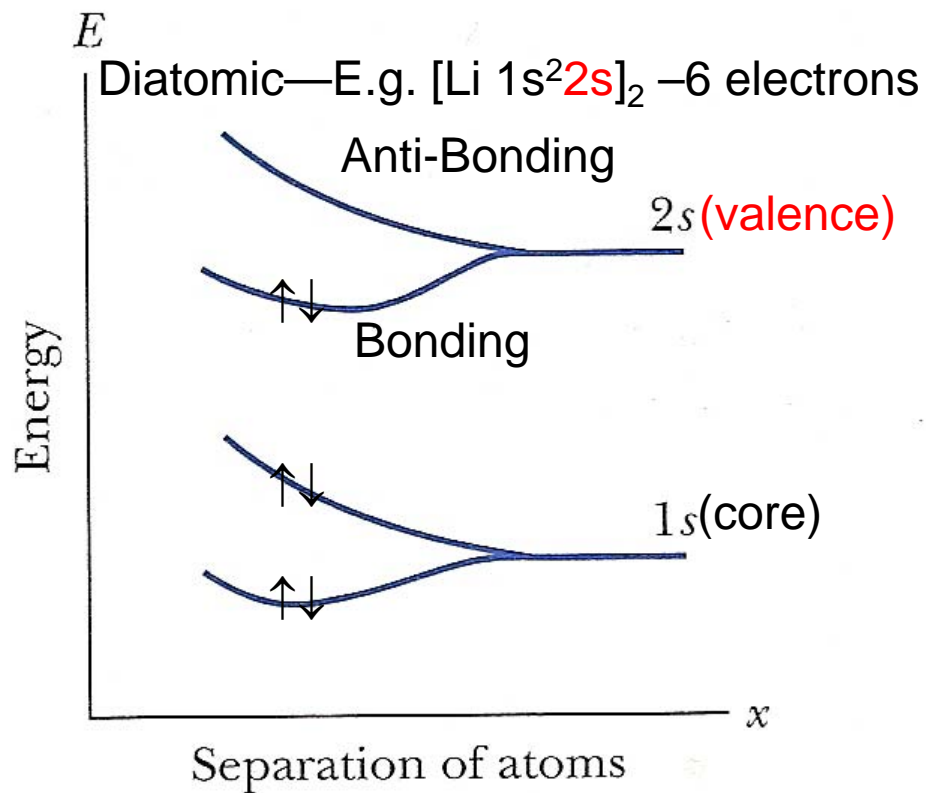
Hydrogen bonding in DNA



Bonding in solids/solid-state devices as an extension of that in molecules



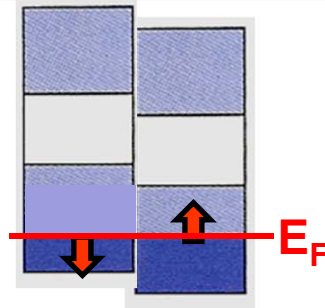
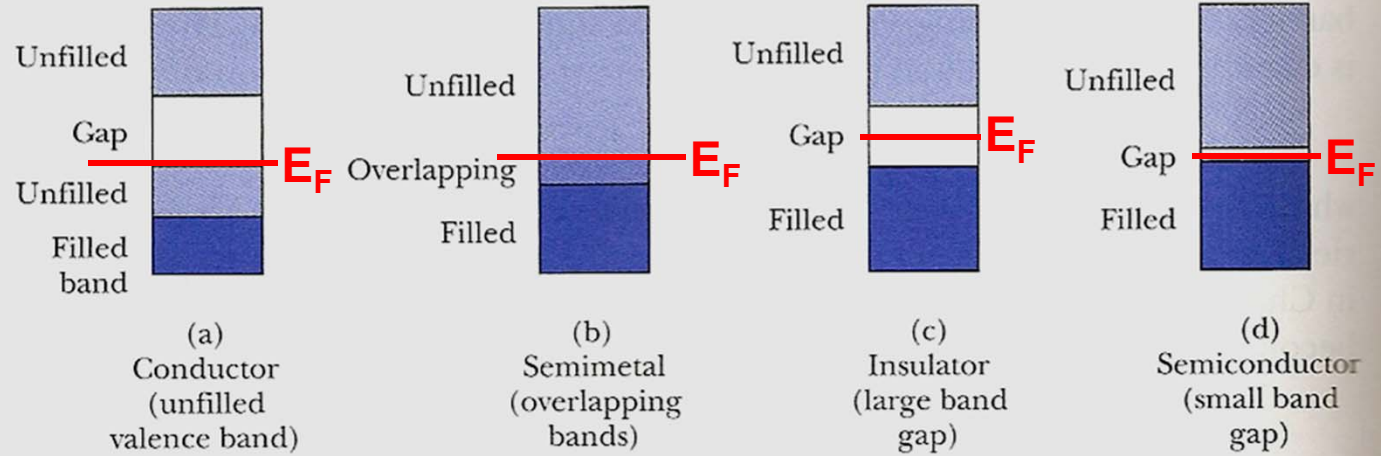
Bonding in solids/solid-state devices as an extension of that in molecules



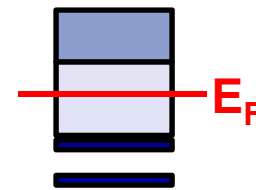
Example: $N = 11$: 22 e^- in 1s-derived
11 e^- in 2s-derived

The types of band structures-by conductive behavior

Figure 11.6 Possible band structures: (a) a conductor with an unfilled valence band, (b) a conductor with overlapping valence and conduction bands (a semimetal), (c) an insulator due to its large band gap, and (d) a semiconductor (due to its small band gap).



Spin-down Spin-up
Ferromagnetic
Conductor
(The exchange interaction)



(f)
Ionic solid:
Very narrow
atomic/ionic
filled bands